Analysis method of internal force distribution on composite section for arch bridge strengthening

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ABSTRACT: It's a common method to strengthen arch bridges by enlarging the section of main arch ring. The composite section mechanically belongs to two-stage loading composite structure, which creates difficulties for easy evaluation of bearing capacity of the new structure. Based on assumption of plane section, an internal force distribution formula on the composite section was deduced. In terms of different stages, the internal force was superimposed for both the old and the new section separately. Meanwhile, the new structure load-bearing capacity was checked. The analysis method has been applied in the strengthening design of arch bridges.

1 GENERAL INSTRUCTIONS

It's a common method to strengthen arch bridges by enlarging sections of the main arch ring, which improves the integrity, stiffness and anti-cracking performance of structural members by increasing their size of sections and reinforcement in order to achieve reinforcement effect to the original structure. The main arch ring, strengthened by enlarged sections, belongs to the composite structure on the mechanical behavior. The original arch ring structure bearing the dead load of both the original bridge and the new section will work as a composite section to bear the external loads (vehicles, temperature, etc.). Therefore, the calculation of bearing capacity can not be mapped simply just according to converted stiffness of the whole section. At present, the methods for analyzing such composite structures mainly include stress superimposion method and displacement compatibility method . Stress superimposion method simulates composite section of new and old arch ring by using the solid element, which can really simulate the stress distribution on the composite section. The displacement compatibility method simulates composite section by using multiple beam elements connected by rigid-arms to ensure the displacement compatibility on the nodes. And also the composite section satisfys plane assumption through nodal displacement coordination. The above two methods are able to analyze the two-stage-loading composite structure from the point of view of between stress superimposition and internal force superimposed, respectively. Based on plane assumption of section, the internal force distribution formulas for the old and newly reinforced sections were deduced. Then the internal forces of the old and new sections were calculated by using the internal force superimposition method. In the end, the sections with different materials are designed and calculated in the light of bearing capacity limit state.

2 INTERNAL FORCE DISTRIBUTION ANALYSIS METHOD OF CO-SECTION

2. 1 Internal force distribution relationships of loaded Co-section together

Under external loads, reinforced arch ring by enlarged sections' method, which its old and new sections deform together and deformation accords with assumption of the plane section.The so-called the internal force distribution of Co-section means is that, when the whole section generate internal force N,M, they can act on old arch section and new section according to a certain scale, internal force is N_1,M_1 and N_2,M_2 ,.See for example Fig.1.

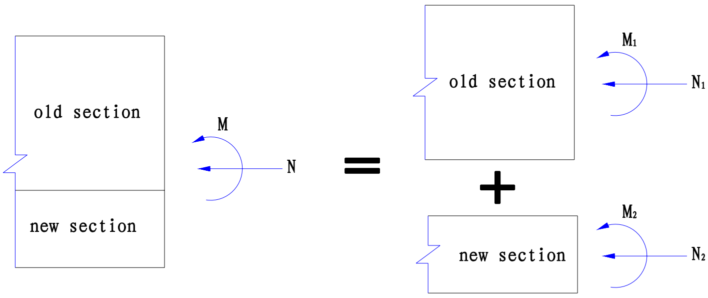

Figure 1 : The sketch drawing of section internal force distribution

According to section internal force equilibrium condition, the relationship between the distributed internal force and the whole section internal force, see for example Eq.1:

$$\begin{cases} M = N_1 y_1 - N_2 y_2 + M_1 + M_2 \\ N = N_1 + N_2 \end{cases} \tag{1}$$

Where y_1 and y_2 are respectively distance from new and old section's heart to Co-section's axis of heart.

To take the plate arch reinforcement for an example(Fig.2), the relationship between the various parts' distribution internal force and section attribute, material property is further analyzed.

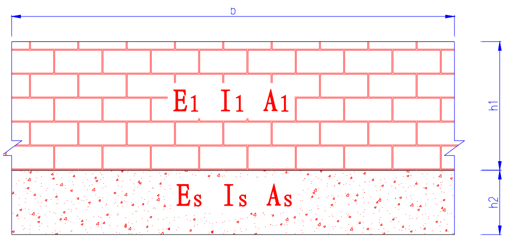

Figure 2 : The structural drawing of the widening section method for reinforcing arch ring

According to the principle of equivalence conversion, the new section material parameters will be converted to the material parameters of the original arch ring as Eq.(2)(3).

$$A_2 = n_e A_s \tag{2}$$

$$b_2 = n_e b \tag{3}$$

where A_2 and A_s is the area of new section and transformed area,b and b_2 is the new arch ring section width and transformed width, the $n_e = E_s/E_1$, is the old and new section ratio of elastic modulus.

After conversion, the area of Co-section and moment of inertia is A and I. According to the plane assumption, under the internal force M and N, the curvature of original arch ring section and new sectionφ_1,φ_2 are equal to Co-section φ, that is,

$$\phi = \phi 1 = \phi \tag{4}$$

According to mechanics of materials formulas can be got as following,

$$\phi = \frac{M}{E_1 I} = \frac{M_1}{E_1 I_1} = \frac{M_2}{E_2 I_2} \tag{5}$$

Finishing Eq.(5) can get moment formula of old and new section distribution.

$$\begin{cases} M_1 = \dfrac{I_1}{I} M \\ M_2 = \dfrac{I_2}{I} M \end{cases} \tag{6}$$

The size of bearing axial force in original section and new section of arch ring depends on the size of strain in various sections neutral axis. The axial force distributed to the sections can be educed by calculating strain of new and old sections in neutral axis.Fig.3 is the strain of new and old sections, under M and N.

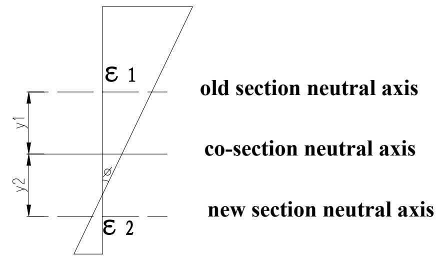

Figure 3 : The strain drawing of Co-section

According to the plane assumption, the strains of the new and the old sections on neutral axis are defined as:

$$\begin{cases} \varepsilon_1 = \dfrac{N}{E_1 A} + \dfrac{M}{E_1 I} y_1 \\ \varepsilon_2 = \dfrac{N}{E_2 A} - \dfrac{M}{E_2 I} y_2 \end{cases} \tag{7}$$

The new and old sections' relationship of the axial force distribution meets relationship formula: $N_i = EA_i\varepsilon_i$, which is used for Eq.(7) and calculates axial force of various of sections as following.

$$\begin{cases} N_1 = A_1(\dfrac{N}{A} + \dfrac{M}{I} y_1) \\ N_2 = A_1(\dfrac{N}{A} - \dfrac{M}{I} y_1) \end{cases} \tag{8}$$

In accordance with Eq.(6) and (8) can obtain the various parts of internal force distribution of Co-section.

2.2 Calculate the internal force distribution of shrinkage and creep

Reinforced arch ring by enlarged sections' method commonly uses concrete material. As the concrete shrinkage and creep effects, Co-section will generate constraint internal force and sub-internal force. The concrete creep and shrinkage deformation of the old bridges nearly completed, but the new section concrete shrinkage and creep have significant effects on internal force distribution.

2.2.1 Calculate the shrinkage constraint internal force

Assumed that the old and new sections can be relatively sliding, in the shrinkage of concrete, connection interface meets the deformation coordination conditions, that is, the old and new sections' deformation of connection interface to meet: $\varepsilon_{j1}=\varepsilon_{j2}$. See for example Fig.4, 1—1 and 2—2 are new and old sections' neutral axis.

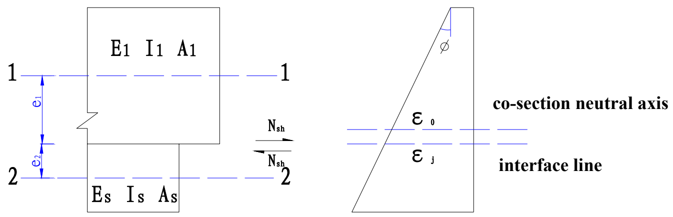

Figure 4 : The calculation sketch drawing of self constraint internal force of arch ring shrinkage

Assumed that the final value of free shrinkage strain of the new concrete section is ε_n , according to the deformation coordination condition can be got as following,

$$\frac{N_{sh1}}{E_1 A_1}+\frac{M_{sh1}e_1}{E_1 I_1}=\varepsilon_n-(\frac{N_{sh2}}{E_s A_s}+\frac{M_{sh2}e_2}{E_s I_s}) \tag{9}$$

According to section internal force equilibrium condition can be got as following,

$$\begin{cases} N_{sh1}-N_{sh2}=0 \\ N_{sh1}e_1+N_{sh2}e_2+M_{sh1}+M_{sh2}=0 \end{cases} \tag{10}$$

Owing to deformation of Co-section accords with the plane-section assumption, that is,

$$\phi=\frac{M_{sh1}}{E_1 I_1}=\frac{M_{sh2}}{E_s I_s} \tag{11}$$

Based on Eq.(10)and Eq.(11),the relationship is,

$$\begin{cases} M_{sh1}=\dfrac{N_{sh1}(e_1+e_2)E_1 I_1}{E_1 I_1+E_s I_s} \\ M_{sh2}=\dfrac{N_{sh1}(e_1+e_2)E_s I_s}{E_1 I_1+E_s I_s} \end{cases} \tag{12}$$

Based on Eq.（9）and Eq.（12）,the relationship is,

$$N_{sh1}=N_{sh2}=\frac{\varepsilon_n}{\dfrac{1}{E_1 A_1}+\dfrac{1}{E_s A_s}+\dfrac{(e_1+e_2)^2}{E_1 I_1+E_s I_s}} \tag{13}$$

2.2.2 Calculate the shrinkage sub-internal force

Base on the deformation compatibility , the strain ε_0 on neutral axis of Co-section and deformation curvature φ as following：

$$\begin{cases} \phi = \dfrac{M_{sh1} e_1}{E_1 I_1} \\ \varepsilon_0 = \dfrac{N_{sh1}}{E_1 A_1} + \dfrac{M_{sh1} e_1 y_1}{E_1 I_1} \end{cases} \tag{14}$$

According to Eq.(14). In accordance with the method of structural mechanics, the sub -internal force M_c and N_c of Co-section can be given. Then according to the described method in 2.1, distributed sub-internal force M_{ci} and N_{ci} respectively of new and old sections will be gained.

2.2.3 With regard to consideration of creep effects

In view of the arch bridge reinforcement structure is special, and the original structure bears most of long-term loads (such as dead load ,etc.), and creep internal force is a small proportion. Therefore, it is the approximate reduction of elastic modulus method to simulate the relaxation effects due to creep. That is,

$$E_s = kE_s' \tag{15}$$

4 ENGINEERING APPLICATION

A stone arch bridge has been built up and used over 20 years, whose bridge roadway width is 8m, span length is 20m, rise-span ratio is 1/4, and the thickness of arch ring is 60cm. Now a large scale equipment will pass through the bridge, so the bridge need be designed to reinforce and its bearing capacity should be checked. The software Midas civil is adopted to carry out the structural analysis and the checking calculation of the bearing capacity of the bridge. And there is the model diagram as the following Fig.6.

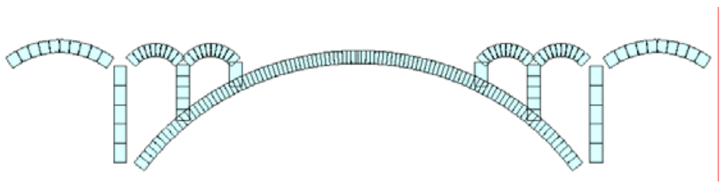

Figure 6 : The FEM model of the original bridge

After the analysis and calculation, it can be see that under the co-action of the dead load and extra heavy load, the vault sagging moment is large, and the section resistance of vault is small to the requests of the Code for Design of Highway Masonry Bridges and Culverts (JTG D60－2005). According to the stress characteristic of the original bridge, the strengthening scheme design is set out that: The main arch ring should be strengthening with cladding reinforcement mat and C40 concrete should be adopted for anchor bolt-spray with thickness of 15cm. The bearing capacity of the bridge after strengthening should be checked with the method in this text. The concrete is strengthened by anchor bolt-spray, so all the dead load (including the dead load of the new section) will be taken by the original arch ring, and the internal force of the dead load in the new arch ring is zero. Refer to the following table 1 for the result of the section internal force in the old and new arch ring.

In accordance with the load combinations defined in the General Code (JTG D60－2004), the internal force combination of the old and the new arch ring section was computed. According to the different material composition, the old and new arch ring section should be checked computations respectively in accordance with the relevant Codes. The results of the checking is shown that the ultimate bearing capacity requirement of the masonry materials should be satisfied by the original arch ring section after strengthening, and bearing capacity requirement of the reinforced concrete materials should be satisfied by the new arch ring section.

Table 1 : The result of the section internal force in the old and new arch ring

Load	Position	Old section		New section		Co-section	
		Axial	M-y	Axial	M-y	Axial	M-y
Original load(1)	Arch springing section	-4072.57	-165.25				
	1/4L section	-2927.9	227.26				
	Vault section	-2683.57	-102.58				
New load(2)	Arch springing section	-443.02	-12.63				
	1/4L section	-350.03	-21.86				
	Vault section	-314.45	32.15				
Shrinkage and creep	Arch springing section	-1419.84	-59.84	1507.98	-5.84		
	1/4L section	-1794.11	-6.54	1964.13	-0.64		
	Vault section	-2213.25	53.9	2489.69	5.26		
Extra heavy load	Arch springing section	-1393.47	-116.36	95.42	-11.36	-1298.06	-460.32
	11/4L section	-985.25	105.97	530.30	10.35	-454.95	419.21
	Vault section	-2377.61	254.65	1258.68	24.87	-1118.93	1007.38

*Note: The units for the axial force and the flexural moment in this table is respectively kN and kN·m, and the axial force is plus when it is tensile and minus in the press.

5 CONCLUSION

With the help of composite section analysis method, the internal force in either the old or the new section can be calculated separately. And the internal force on the composite section can be calculated easily through the method of internal force distribution. Then the resisting force can be calculated in accordance with the corresponding standards or codes in the light of different materials. Practice has proved that this method is simple, reliable and applicable to the internal force calculation for arch ring strengthening design.

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