Evaluation of the seismic response of masonry arch bridges modeled using beam elements with fiber cross section

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ABSTRACT: An approach for the evaluation of the seismic response of masonry arch bridges is proposed and applied to an existing multi-span railway viaduct. Aiming at performing non-linear analyses with low computational costs, a 1-D modelling strategy which makes use of beam elements with fiber cross section is developed. First of all, the failure condition of a single arch subjected to an acceleration pulse at the base is analyzed and the results are compared with the solution provided by the mechanism method. Then, an historic bridge is modelled: in a first step, its dynamic behavior in the elastic range is compared with that of a 3-D finite element model to verify the reliability of the fiber beam approach for what concerns both natural frequencies and modal shapes. Finally, the seismic response of the bridge is evaluated by means of non-linear push-over and dynamic analyses under suitable natural accelerograms.

1 INTRODUCTION

Masonry arch bridges are still nowadays an important element of the Italian and European infrastructural net; their assessment with respect to seismic loads is often needed, considering also the increased safety standard requested by the actual codes. Nonetheless, their dynamic behaviour is, in great measure, still unexplored.

The dynamics of masonry arches has been primarily investigated making use of the mechanism method (Oppenheim 1992; Clemente 1998; De Luca et al. 2004), by establishing the maximum ground acceleration the structure can sustain; it means that the evaluation of the safety level towards earthquakes consists in verifying that the expected PGA is not higher than the limit value turning the structure into a mechanism. This formulation, which is suitable for elegant analytical solutions, has been compared with numerical simulations performed using distinct element models by DeJong and Ochsendorf and De Lorenzis et al.. These approaches are usually based on the classical hypothesis that the material has no tension resistance and infinite compression strength (Heyman 1966); the latter assumption, however, may lead to an over-esteem of the effective structural safety level (de Felice 2009).

Recently, some studies have been developed making use of 3-D elasto-plastic or 1-D non-linear finite elements (Resemini and Lagomarsino 2007) which allow to assess the seismic safety throughout a performance based approach, coherently with the most advanced regulations, or to evaluate the seismic fragility, as proposed by Carbone and de Felice, making use of the response surface method.

In the present work the possibility of using fiber beam elements for the seismic response evaluation of masonry arch bridges is investigated. The developed modelling strategy is suitable for performing non-linear static and dynamic analyses with low computational costs and allows to take into account the effective mechanical behaviour of historic brickwork under cyclic eccentric compression (de Felice and De Santis 2010), which is the stress condition experimented by piers and vaults sections (de Felice 2009).

First of all, the dynamics of a single arch under constant base acceleration is investigated considering an ENT rigid material, and the predicted failure condition is compared with the solution provided by the mechanism method to verify the reliability of the proposed approach. Then, an existing viaduct belonging to an Italian railway line built at the end of XIX Century is modelled; it can be taken as a representative example of the several bridges realized in Italy in the same period, showing similar geometric characteristics, material properties and building techniques. The dynamic behaviour of the bridge in the elastic range is examined by means of a linear 3-D finite element model, and its natural frequencies and modal shapes are compared with the results of the fiber beam model, to have a further validation confirm. Finally, non-linear push-over and dynamic analyses are performed, and their predictions are compared in terms of resultant base shear and displacement response.

2 COLLAPSE OF MASONRY ARCHES SUBJECTED TO IMPULSE BASE MOTION

To assess the capability of a modelling strategy which makes use of non linear beam elements with fiber cross section, a sample problem of an arch (Fig.1) under impulse base motion is considered, and the maximum sustainable acceleration a for a given impulse duration ti is evaluated by means of non-linear dynamic analyses. The validation of the proposed approach is made by comparison with the solution provided by the mechanism method, developed by Oppenheim and recalled by De Lorenzis et al..

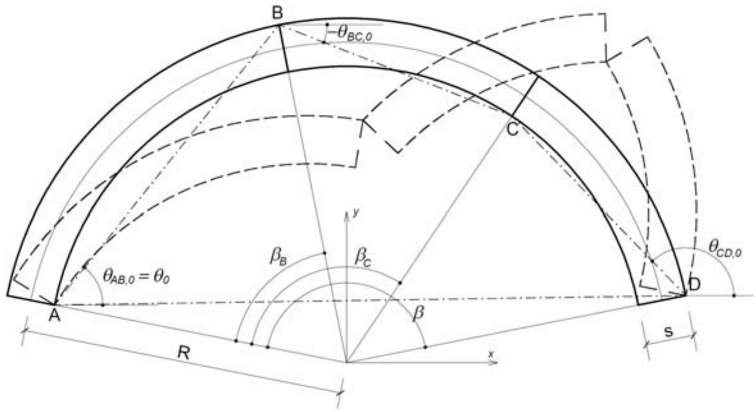

Figure 1 : The circular arch as a four-bar linkage mechanism in its initial and deformed configurations.

The examined problem is solved only for the first half cycle of motion; according to the mechanism method assumptions, masonry is considered rigid, with infinite compressive strength and no tensile resistance; finally, the positions of the plastic hinges (identified by the angles β_B and β_C) are assumed a priori. Under these conditions the arch is a mechanism formed by the four-bar linkage ABCDA, and has only one degree of freedom, i.e. the deformed configuration, described by the rotations of the links, depends on only one of them (in this case $\theta_{AB}=\theta$), arbitrarily chosen as the Lagrange parameter of the system. The rotations θ_{BC} and θ_{CD} and rotational velocities $\dot{\theta}_{bc}$ and $\dot{\theta}_{cd}$ are written in terms of θ and $\dot{\theta}$ through displacement and velocity analyses (Erdman and Sandor 1984).

By doing so, θ is the unique parameter of Hamilton's principle (1), in which $V(\theta)$ is the potential energy, $T(\theta, \dot{\theta})$ is the kinetic energy and $Q(\theta)$ is the forcing function, expressing the work variation performed by the external force $a \cdot (m_{AB} + m_{BC} + m_{CD})$, being the terms into brackets the masses of the links AB, BC and CD, respectively. The equation of motion (2), containing the radius of the middle surface R and the gravity acceleration g, depends on the coefficients $M(\theta)$, $L(\theta)$, $F(\theta)$ and $P(\theta)$, which depend on the geometry of the system and are strongly non-linear in θ. Eq. (2) is solved assuming constant values of the coefficients, evaluated in the initial geometry in which $\theta(t = 0) = \theta_0$, leading to a tangent approximation of the response, that is licit in the small rotations field (3).

$$\frac{\partial}{\partial t}\left(\frac{\partial t}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}+\frac{\partial V}{\partial \theta}=Q \tag{1}$$

$$M(\theta)R\ddot{\theta}+L(\theta)R\dot{\theta}^2+F(\theta)g=P(\theta)a \tag{2}$$

$$\theta(t)=\frac{M}{L}\ \log\left(\cos\left(\frac{\sqrt{L\ (F\ g-P\ a)}}{M}t\right)\right) \tag{3}$$

In the present case, an arch identical to the one investigated by Oppenheim having radius R = 10m, thickness s = 0.15R, and angle of embrace β= 157.5° is considered; the positions of the plastic hinges are defined a priori by the angles β_B= 67.5° and β_C= 112.5° (Fig.1).

The variation of the potential energy $V(\theta)$ versus θ is plotted in Fig. 2a; it grows between θ_0 and θ_1 and then decreases; it is noteworthy that the maximum value $V(\theta_1)$ is reached after a very small rotation (0.07rad) about the initial geometry (note that during motion θ decreases).

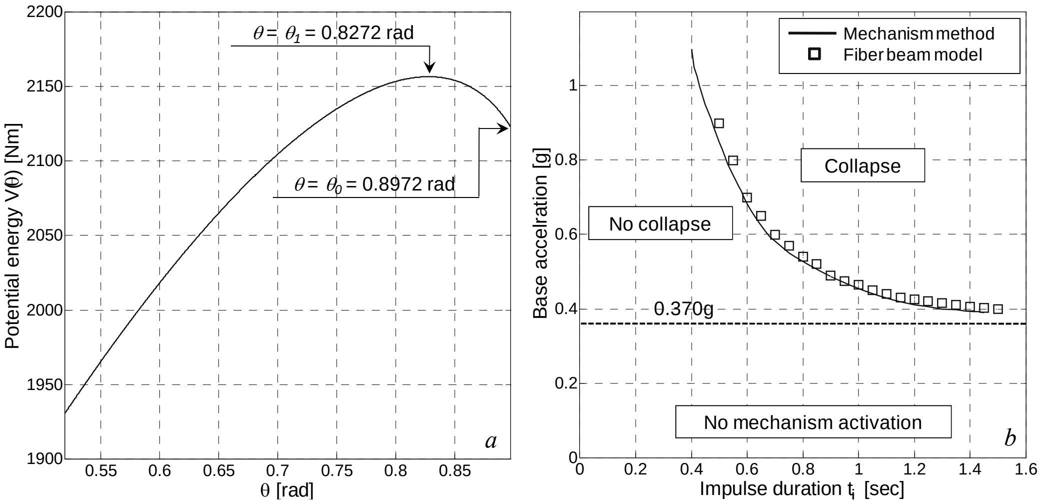

Figure 2 : Potential energy (a) and failure domain under impulse base motion (b) for the examined arch

The arch fails as soon as the total work done by the inertial forces in the duration t_i is equal to the difference in potential energy between θ_1 (non-recovery point) and θ_0 (initial geometry), as stated by (4), in which $v_{AB}(\theta(t))$, $v_{BC}(\theta(t))$, and $v_{CD}(\theta(t))$ are the horizontal components of the velocities of the link centers of mass:

$$\int_{t_0=0}^{t_i} a\ \left(m_{AB}\ v_{AB}(\theta(t))+m_{BC}\ v_{BC}(\theta(t))+m_{CD}\ v_{CD}(\theta(t))\right)\ \mathrm{d}t=V(\theta_1)-V(\theta_0) \tag{4}$$

It has to be pointed out that this procedure is perfectly equivalent to the one developed by Housner (1956) to identify the overturning of a rigid block under constant acceleration.

Eq. 4 is numerically solved by substituting $\theta(t)$ given by (3), and the curve plotted in Fig. 2b is obtained, representing the collapse impulses for the considered arch. For short impulse durations a high acceleration is needed to induce the structural collapse, while for $t_i \rightarrow \infty$ the curve tends asymptotically to a limit value a = 0.370g. Three domains can be identified in the graph, as it is pointed out also by Clemente: if a < 0.370g the horizontal acceleration is not even sufficient to turn the arch into a mechanism, as no hinging occurs; on the contrary, when $a \geq$ 0.370g the onset of motion takes place. If the point (t_i, a) is below the curve, then it represents an impulse which does not cause the structural collapse, i.e. there is hinging but the arch returns to its initial geometry; finally, if the couple (t_i, a) identifies a point above the curve, then the corresponding impulse causes the arch failure.

The static multiplier $a / g = 0.370$ can be easily found as the ratio between the virtual works of horizontal and vertical loads, that is equal to $F(\theta_0) / P(\theta_0)$, as stated in (2) if dynamic effects are neglected.

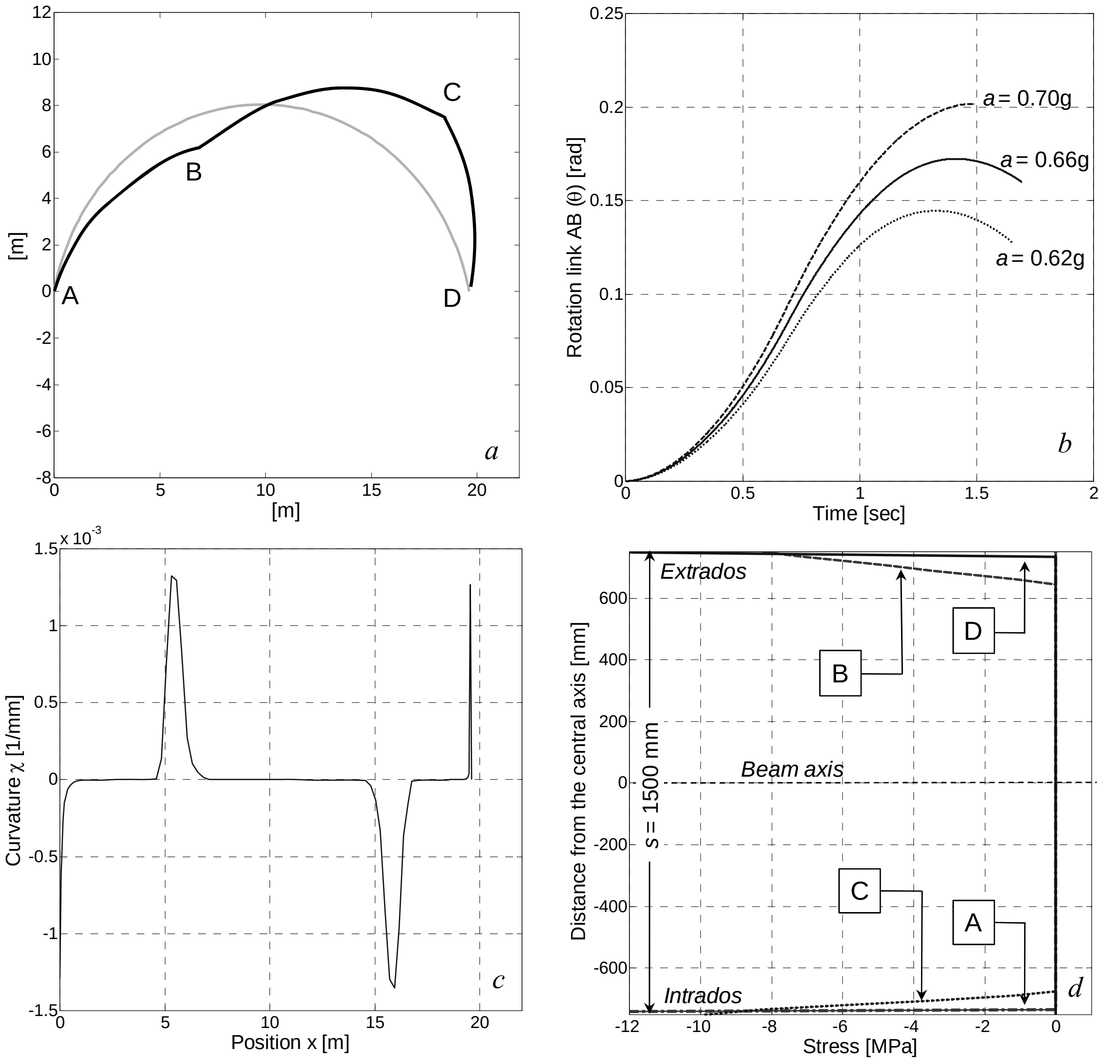

Figure 3 : Fiber beam model results: collapse configuration (a), first link rotations for different values of base acceleration (b), elements curvature (c) and stress field in the hinge sections (d), for $t_i = 0.70$ sec.

Let us now consider the examined arch as a segmental beam, made by 100 non-linear finite elements with fiber cross section (Spacone et al. 1996); the investigation of its failure under impulse base motion is performed by systematically repeated analyses, carried out for several impulse durations so as to find as many collapse acceleration values. Aiming at reproducing the same assumptions made by the mechanism method, no damping is considered and the material assigned to the fibers has no tension resistance and infinite compression strength. No plastic hinges are defined a priori to check if the model is able to predict a correct collapse mechanism. The result is found to be independent on the number of beam elements or of fibers in their cross section, provided that an adequate discretization is ensured, that does not affect the geometry of the arch (inadequate number of elements) or the stress distribution within the cross section (inadequate number of fibers).

As can be seen in Fig.2b, in which numerical simulations are represented by the square marks, a very good agreement between fiber beam model and mechanism method is found, apart from very short impulses (ti < 0.5 sec) for which no stable solution is obtained.

The collapse is assumed to occur when the rotation of the first link AB diverges, i.e. does not go back to zero after the rotation peak (Fig.3b), and no equilibrium solution is found. The failure configuration, characterized by the activation of a four-hinge mechanism, is correctly predicted (Fig.3a), together with the position of the hinge sections identified by the peaks in the curvature diagram (c), where a slight spreading is due to the continuum nature of the modelling approach; the stress field is plotted in Fig.3d for sections A, B, C, D, pointing out the level of partialization of the four hinges, in which the load resultants are next to the section edge (intrados and extrados, alternatively).

3 DESCRIPTION OF THE BRIDGE UNDER STUDY

The bridge under study is Ronciglione Viaduct, the most important work of the railway line Rome-Viterbo and built between 1890 and 1894. It has a rectilinear layout and a very slender aspect, and is made of seven barrel vaults and six piers, with a maximum height of about 45m (Fig. 4).

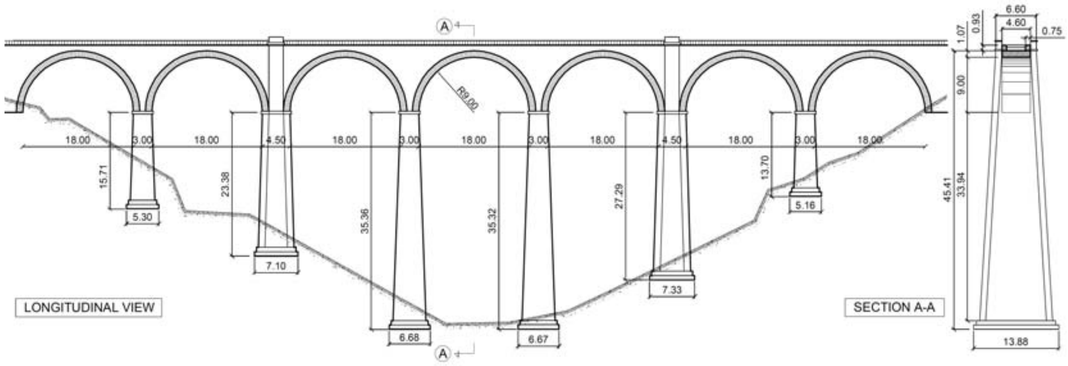

Figure 4 : Longitudinal view and transversal section of Ronciglione Viaduct.

The masonry of the piers is in rough tuff stone, with squared stones on the external face; the second and the fifth piers are provided with buttresses in transversal direction and are dimensioned 1.50m larger in longitudinal direction; all the piers have a vertical slope of 3.5%. The barrel vaults are made with clay bricks, having dimensions 28×14×6cm3, and hydraulic mortar consisting of lime and pozzolana, without cement; the vaults have 18.00m span, 9.00m rise, 1.07m thickness and 4.60m depth. According to the original drawings, the spandrel walls are 75cm thick and 11.00m high from the springing, and are made of regular courses of tuff squared stones, while the backfill height is about 4.70m from the springing. The bridge is not in service any more, but is still in good maintaining conditions.

4 DYNAMIC CHARACTERIZATION

The dynamic behaviour of Ronciglione Viaduct in the elastic range is investigated by a linear 3-D finite element model, made of brick elements with 8 nodes (Fig.5). The mechanical properties assigned to the brickwork of the vaults (Young modulus E = 2500MPa and self weight γ = 1650kg/m^3) are derived from experimental tests (de Felice and De Santis 2010), while E = 2500MPa, γ=1500 kg/m^3, and E = 200MPa, γ = 1500kg/m^3 are chosen for the piers tuff masonry and for the fill soil, respectively. Sensitivity analyses are performed to investigate how the material characteristics influence the structural response, and strong dependence on the masonry of the piers is found because of their height of the bridge.

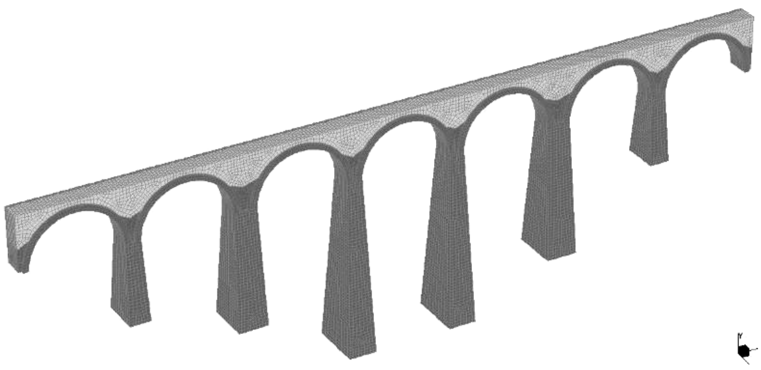

Figure 5 : Model of Ronciglione Viaduct with brick elements.

The bridge conformation, characterized by high central piers, is such that the principal modal shape is in transversal direction and nearly symmetric, with all nodal displacements of the same sign and higher for the central spans; the following mode having a significant contribution in terms of participating mass is the fourth one, that is the first mode in longitudinal direction (Fig. 6).

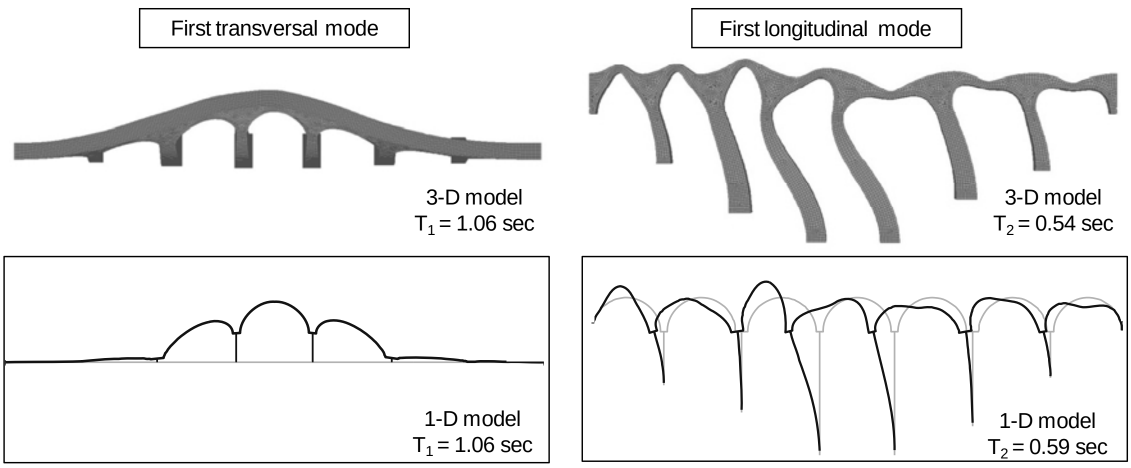

Figure 6 : First modal shapes in the transversal (left) and longitudinal (right) directions.

The results achieved from the 3-D model are compared with a 1-D model, in which the vaults and the piers are described using fiber beam elements, while for the backfill, the spandrel walls and the abutments truss elements with fiber cross section are used; the fill soil mass is represented through point-masses connected by rigid links to the underlying vaults. Totally, 50 beams are used for each pier, and 100 for each arch, 17 truss elements describe each backfill, and 34 are adopted for each spandrel wall and each abutment. The explicit description of the spandrels is essential to get a satisfactory representation of the dynamic behaviour of the bridge, as pointed out by Fanning et al. and Brencich and Sabia. In this phase, the same mechanical properties adopted for the 3-D model are assigned to the fibers of the 1-D one. A very good agreement is found in terms of natural frequencies as well as of modal shapes (Fig. 6), for both transversal and longitudinal directions: in the former case an identical natural period T1 = 1.06sec is obtained by 3-D and 1-D models, while in the latter one the two approaches lead to very close values: T2 = 0.54sec and T2 = 0.59sec for 3-D and 1-D model, respectively; this small difference is probably due to the different description of the spandrels.

5 NON-LINEAR STATIC ANALYSES

Push-over analyses are carried out on Ronciglione Viaduct under in-plane and out-of-plane horizontal loads, with two different distributions: the first one is proportional to nodal masses, while the second one is proportional to nodal masses times horizontal displacements of longitudinal and transversal principal modes. The effective brickwork properties are described by adequate non-linear constitutive laws (de Felice and De Santis 2010). Fig.7 shows the capacity curves in which the control displacements (the springing and the key node of the central span, for the different planes) is plotted versus the resultant base shear Vb normalized for self-weight W, pointing out a lower resistance towards transversal forces and a great dependence of the load distribution on the maximum load the bridge can sustain; for the second distribution (push-over #2) a maximum base shear equal to 0.18 W and 0.09 W is found for longitudinal and transversal directions respectively. The sensitivity of out-of-plane capacity curves to the variation of brickwork mechanical properties is shown in Fig.8.

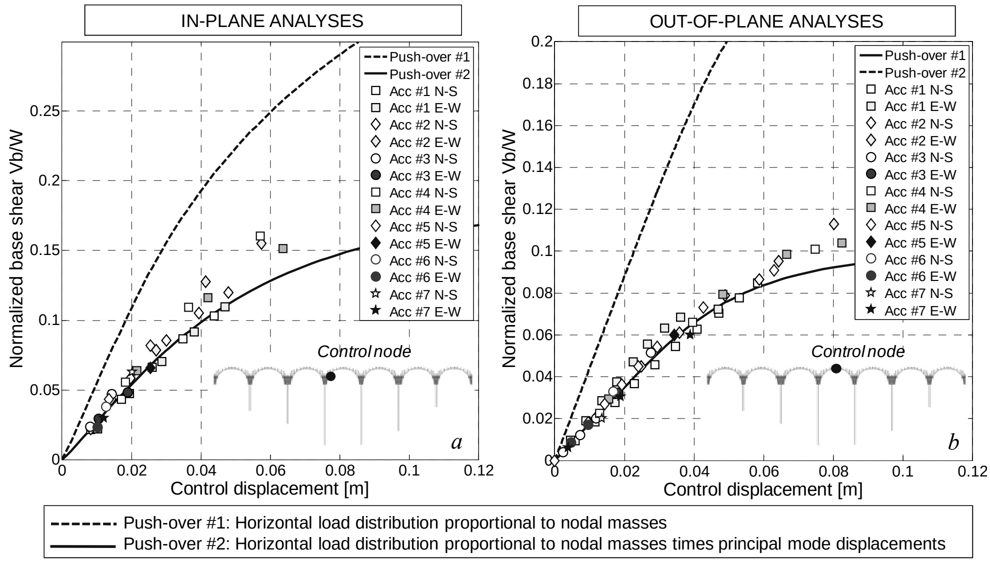

Figure 7 : Capacity curves for different horizontal load distributions and dynamic simulations predictions for in-plane (a) and out-of-plane (b) analyses.

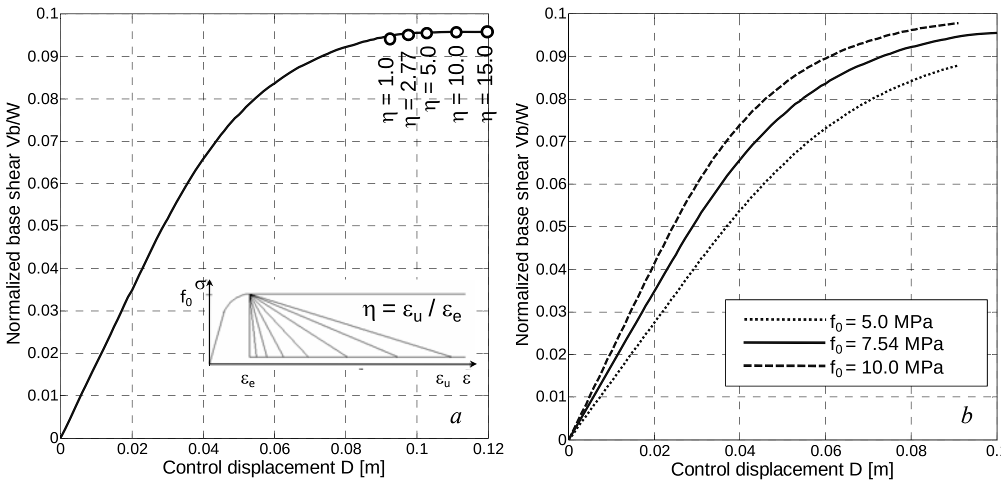

Figure 8 : Dependence of out-of-plane capacity curves on softening branch slope (a) and compressive strength (b).

6 NON-LINEAR DYNAMIC ANALYSES AND COMPARISONS

The seismic response of Ronciglione Viaduct is evaluated by means of non-linear dynamic analyses, performed under seven natural accelerograms. The records are selected within the European Strong-Motion Database (ESD), considering only European earthquakes and stations with both North-South and East-West components available. Input signals are chosen among events having moment magnitude at least equal to 5.8, and so that their average elastic spectrum is close to the synthetic spectrum provided by the code for the site (the spread is lower than 10% in the 0.15 ÷ 2.00sec range).

The application of each signal is repeated several times, with growing scaling factor, and for each simulation the maximum resultant base shear (Vb) and the corresponding control displacement (d) are recorded, so as to obtain a set of couples (d, Vb), useful for the comparison with push-over analyses. It is seen from graphs in Fig. 7 that the points representing dynamic simulations retrace with good approximation the capacity curves, if a horizontal load distribution proportional to nodal masses times principal mode displacements (push-over #2) is applied; for high scaling factors no equilibrium solution is found.

On the whole, despite some small differences and considering the limits deriving from the reduction to an equivalent SDOF system, static analyses seem to provide reliable results for the examined bridge, which is characterized by an evident symmetry and a considerable height of the central piers, making the dynamic response mainly governed by the first mode and reducing the approximations of the push-over method assumptions. Clearly, these considerations need to be supported by a more extended number of applications.

REFERENCES

Brencich A., Sabia D. 2007. Tanaro bridge: dynamic tests on a couple of spans. Bridge Engineering 12(5): p.662-665.

Carbone I., de Felice G. 2007. Fragilità di ponti ad arco in muratura sotto azioni sismiche longitudinali. Proc. XII Convegno ANIDIS, Pisa, Italy, 10-14 June (in Italian).

Clemente P. 1998. Introduction to dynamics of stone arches. Earthquake Engineering and Structural Dynamics, 27 p.513-522.

De Felice G. 2009. Assessment of the load-carrying capacity of multi-span masonry arch bridges using fiber beam elements. Engineering Structures 31(8): p.1634-1647.

De Felice G., De Santis, S. 2010. Experimental investigation and numerical modelling of historic brickwork for railway arch bridges assessment. Int. Journal of Architectural Heritage 4(2): p.115-137.

De Lorenzis L., DeJong M., Ochsendorf, J. 2007. Failure of masonry arches under impulse base motion. Earthquake Engineering and Structural Dynamics 36, p.2119-2136.

De Luca A, Giordano A., Mele, E. 2004. A simplified procedure for assessing the seismic capacity of masonry arches. Engineering Structures 26, p.1915-1929.

De Jong M., Ochsendorf J. A. 2006. Analysis of vaulted masonry structures subjected to horizontal ground motion. Proc. 5th Int. Conf. on Structural Analysis of Historical Constructions SAHC'06, New Delhi, India, 6-8 November, p.973-980.

Erdman A., Sandor G. 1984. Mechanism design: analysis and synthesis.: Englewood Cliffs, NJ: Prentice-Hall Inc.

Fanning P. J., Boothby T. E., Roberts B. J. 2001. Longitudinal and transverse effects in masonry arch assessment. Constructions and Buildings Materials 15(1), p.51-60.

Heyman J. 1966. The stone skeleton. Int. Journal of Solids and Structures 2, p.249-279.

Oppenheim I. J. 1992. The masonry arch as a four-link mechanism under base motion. Earthquake Engineering and Structural Dynamics 21, p.1005-1017.

Resemini S., Lagormarsino, S. 2007. Displacement-based methods for the seismic assessment of masonry arch bridges. Proc. 5th Int. Conf. on Arch Bridge ARCH'07, Madeira, Portugal, 12-14 September, p.441-450.

Spacone E., Filippou F. C., Taucer F. F. 1996. Fiber beam-column model for non-linear analysis of R/C frames. Part 1: formulation. Earthquake Engineering and Structural Dynamics 25, p.711-725.