COMPARISON OF VARIOUS MODELLING TECHNIQUES APPLIED IN ANALYSIS OF MASONRY ARCH BRIDGES

C. Costa[1], T. Kamiński[2]

[1]Polytechnic Institute of Tomar, Engineering Department, Tomar, Portugal.
[2]Wrocław University of Technology, Faculty of Civil Engineering, Wrocław, Poland.

e-mails: c.costa@ipt.pt, tomasz.kaminski@pwr.edu.pl

SUMMARY

Structural analysis of masonry arch bridges has been a research topic or a subject or engineering work for a long time. Therefore there is a large number of different approaches to the problem including both analytical and numerical methods. Even only within the group of numerical methods many various procedures and modelling strategies are being applied. In the paper three different approaches are used and compared in a case study of the São Lázaro bridge: two of them using Finite Element Method, differing in modelling techniques of the masonry arch barrel (called micro- and mezo-modelling), and one based on Discrete Elements. Material parameters of the models are based on laboratory tests performed on components of real masonry bridges. Within the analyses the static response to exploitation loads as well as the ultimate load values and failure modes of the bridge are determined. Some conclusions are drawn from the presented analysis as well as further recommendations related to application of the considered approaches to other cases of masonry arch bridges are given.

Keywords: *Masonry, arch bridge, FEM, DEM, load carrying capacity.*

1. EXPERIMENTAL DETERMINATION OF MATERIAL PROPERTIES

1.1. The scope of tests

Laboratory tests of materials corresponding to the ones composing the analysed masonry arch bridge included mechanical and physical examination of granite stone, mortar and backfill soil as well as interaction between the materials. The tested samples were extracted from the old Lagoncinha and Zameiro bridges as well as the new Vila Fria bridge [1] composed of representative materials for Portuguese medieval bridges. Therefore the defined properties could be then applied to numerical models of the bridge analysed in this study.

All the tests (presented in Fig. 1) have been carried at Faculty of Engineering of the University of Porto (FEUP) according to current standardized tests. Detailed collection of the scope of the performed tests with origins of samples is included in Tab. 1.

The granite blocks underwent compression (Fig. 1a) and tensile tests (Fig. 1b). The latter one was carried out by means of Brazilian test. An influence of possible saturation of stone on reduction of its compressive strength was also analysed.

Table 1. Scope of the performed material tests.

Tested material	Evaluated parameters	Origin of samples
Granite stone	- uniaxial compressive strength (Fig. 1a) - tensile strength (Brazilian method, Fig. 1b) - Young modulus, unit weight - porosity, water absorption	Lagoncinha bridge Vila Fria bridge
Mortar	- flexural strength - compressive strength (Fig. 1c)	Vila Fria bridge
Backfill	- oedometric modulus of elasticity - mechanical parameters (triaxial tests, Fig. 1d)	Zameiro bridge Vila Fria bridge
Stone-to-stone & stone-to-infill joints	- Shear strength, elastic shear stiffness (Fig. 1e, f) - compressive strength, normal stiffness	Vila Fria bridge

The mortar was tested at two values of mixing water percentage: 14 and 22%. The mortar deformability (tangent) modulus has been determined based on the stress-strain curve observed in the compression test, corresponding to the range values of 10 to 30 % and 30 to 60 % of the maximum compressive stress (E10-30 and E30-60, respectively).

The compression tests of block-mortar joint assemblages have been performed on a mechanical press with capacity up to 2700 kN. The displacements have been measured by LVDT transducers which have been placed on the four faces of the sample in order to measure the vertical displacement of the mortar joint including a thickness of 17 mm (Fig. 1c).

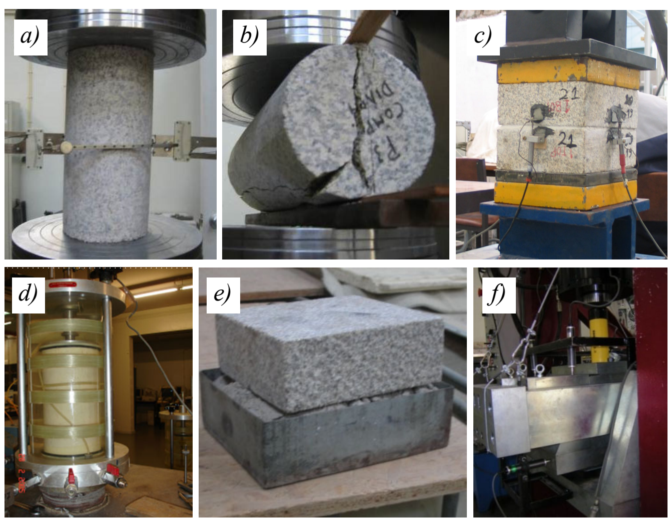

***Fig. 1.** Tested specimens of the tested materials (acc. to Tab. 1).*

For the evaluation of the properties of the backfill materials, triaxial tests have been carried out (Fig. 1d) on samples taken from pure granular soil and mixture of cement and granular soil as actually used in the lower and upper zones of the backfill of the Vila Fria bridge, respectively. The triaxial tests have been performed considering different values of the consolidation stress in order to reproduce the conditions verified on the bridge. Samples of granular infill for oedometric testing extracted from Zameiro bridge included disturbed samples reconstituted in laboratory using the usual procedures in view of the material condition observed in-situ.

Shear tests of block-mortar joint assemblages, joints between blocks and infill material have been performed using shear box (Fig. 1e, f) of 200 mm x 200 mm x 150 mm. The test started with a phase of vertical preloading of various levels, which was held constant during the test, and then the shear force was applied triggering the sliding of the joint.

1.2. Results of the tests

The determined average experimental values of physical and mechanical parameters of the materials coming from the analysed masonry bridges are given in Tab. 2. It was noted that for specimens of Vila Fria bridge the saturation of stone influenced reduction of its compressive strength. The increase in the percentage of water *w*, leads to a decrease in the flexural strength of mortar by 46% and its compressive strength by 12%, while the deformability modulus slightly increases by about 7%.

Table 2. *Physical and mechanical parameters of bridge tested materials.*

Material			**Stone blocks**		**Infill**		**Mortar**	
Bridge of sample origin			**Vila Fria**	**Lagon-cinha**	**Vila Fria**	**Zame-iro**	**Vila Fria**	
							***w*=14 %**	***w*=22 %**
Compressive strength		[MPa]	66 (32.4)*	51.0	-	-	6.9	6.1
Tensile strength		[MPa]	3.7	5.4	-	-	-	-
Bending strength		[MPa]	-	-	-	-	1.7	0.9
Young modulus		[MPa]	22400	39200	30.2	6.25-23.7	-	-
Deformability modulus	E10-30	[MPa]	-	-	-	-	412	408
	E30-60	[MPa]	-	-	-	-	931	993
Unit weight		[kN/m^3]	24.1	26.4	-	-	19.2	18.9

* saturated specimen

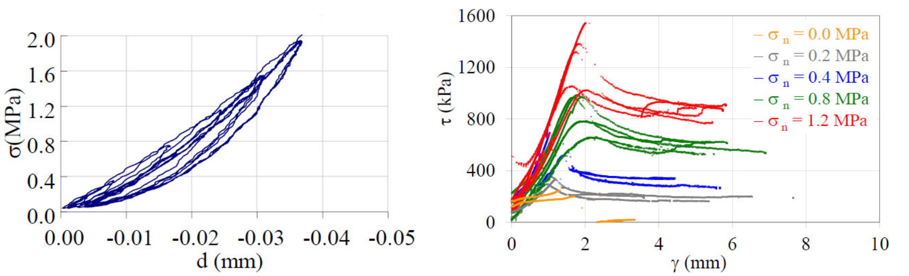

Fig. 2. *Stress-displacement relationships for compressive (left) and shear tests (right) of stone-to-mortar joints.*

Selected results in a form of stress-displacement relationships for compressive and shear tests of stone-to-mortar joints are shown in Fig. 2.

2. ANALYSIS OF THE SÃO LÁZARO BRIDGE

2.1. Description of the structure

The analysed bridge is representative for such Portuguese structures constructed in the medieval period [2]. It is made of granite stone and the main span is formed by a 7.5-metre semi-circular arch. General view of the bridge in shown in Fig. 3.

Fig. 3. *General view of the Lazaro Bridge.*

2.2. Numerical models of the bridge

The simulation of the bridge behaviour was based on 2D FE and DE models, aiming at representing the bridge behaviour in the longitudinal direction. Four various models presented in Fig. 4 were created by means of the following computer software: a) CAST3M (discontinuous FEM), b) ABAQUS (continuous FEM), c) UDEC (DEM) and d) RING (rigid blocks analysis). Values of the material parameters applied in the constitutive models are based on results obtained both from laboratory tests presented in chapter 2.

In the discontinuous FE model (FE/DC) the masonry bridge components (arches and pavement) are represented by means of micro-modelling technique using solid elements to define individual blocks and zero thickness joint elements at their interfaces (stone-to-stone joint type). In the continuous FE model (FE/CT) the joints of the masonry arch are directly represented with solid elements of 7 mm width. In the both models the backfill is represented by means of solid elements connected by zero thickness joint (contact) elements in the interfaces between the infill and blocks of the arch barrel. Specific characteristics for the infill-to-stone joint type are defined below.

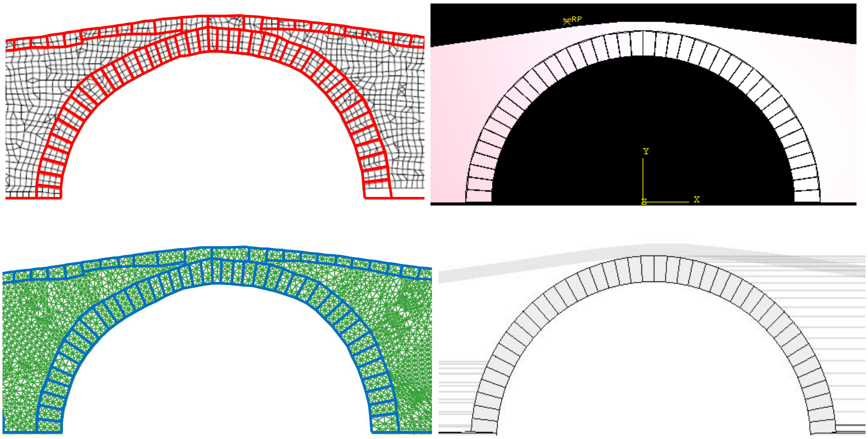

Fig. 4. *Applied models: a) FE/DT, b) FE/CT, c) DE and d) RB models.*

The linear elastic behaviour of stone blocks, mortar and infill material is characterized by the elastic modulus (E) and Poisson's ratio (v) and specific weight (γ) listed in Tab. 3. The table includes also the values of normal (k_n) and shear (k_s) stiffness of the joint elements applied in FE-DC as well as DE models. These values have been based on both the results obtained from laboratory tests and modal identification described in [2].

Table 3. *Elastic parameters of materials applied in FE and DE models.*

Material	E [MPa]	v [-]	γ [kN/m^3]	Material	k_n MPa/mm]	k_s [MPa/mm]
Stone blocks	15500	0.20	26	Stone-to-stone joints	6.24	0.56
Mortar	44	0.17	26			
Infill	30.2	0.3	21.5	Stone-to-infill joints	0.53	0.28

In the FE/DC model the values adopted for the constitutive parameters of joints were obtained from compression and shear tests. The shear strength of joints is determined according to the Mohr-Coulomb failure surface, which is defined by the friction angle ϕ (35.8º for stone-to-stone joints and 33.2º for stone-to-infill joints) and zero cohesion for both joint cases. In the normal direction unlimited strength at compression is assumed and zero tensile strength.

In the FE/CT model stone-to-stone joints are represented by inelastic mortar. It is described by means of elastic-brittle material model with *Concrete Damaged Plasticity* formulation of degradation [3], unlimited compressive strength and some tensile strength with pick value f_t = 10 kPa assumed to provide numerical stability of the solution. Refined stress-strain relationships for tension softening is also defined assuming drop of stresses to about 1 kPa. The stone-to-infill joints are applied according to the same Mohr-Coulomb model as in case of FE/DC however so call “hard” contact is defined what determines rigid connection between contacting materials prior to reaching the failure surface by normal and shear stresses.

The infill material in both FE models is simulated using the Drucker-Prager model considering an elastic-plastic behaviour with 49º friction angle, 17 kPa for cohesion and 10º dilatancy angle.

In the DE model the masonry stone blocks as well as the infill material are defined using discrete deformable blocks, internally discretized into finite triangular elements, with the joints simulated as contacts between the discrete blocks.

The material behaviour of the DE model is defined in agreement with the FE constitutive modelling, including the material parameters included in Tab. 3 and constitutive models to control the nonlinear evolution.

The FE and DE model's geometry including contacts' updating is recomputed at each step of calculations taking into account the bridge response with large displacements and strains. The boundary conditions in FE models are set using rigid supports to fix the displacements at the bridge base. The horizontal displacements of the infill elements are blocked on vertical boundaries at both side edges of the modelled infill area. The fixed displacement directions of the boundary conditions are defined within the DE model as the velocity directions set to zero.

The RB analysis is performed considering identical characteristics for the geometry, materials and loading as for those used in the FE and DE models. The material parameters have been also defined in the way allowing comparison of results coming from the RB model with those obtained from the FE and DE models including friction angle for stone-to-stone and stone-to-infill joints represented by tan ϕ = 0.72 and 0.53, respectively. The infill material modelled within the RING software by means of factors defining passive zones' behaviour somewhere between the lateral earth pressure at rest to the passive lateral earth pressure obtained from the Rankine theory, were established on the way of sensibility analysis [2].

2.3. Analysis procedure

The analysis is carried out in two consecutive steps: in the first one self-weight of the structural components is applied and in the second one an external P load is added. The external load is simulating a vehicle defined in the Portuguese standard RSA and is represented by three vertical axel loads spaced every 1.5 m.

Within the discontinuous FE and DE models the live load is applied incrementally by means of force-controlled procedure with three equal force values at each axle. Within the FE/CT model the load process is controlled by vertical displacement u_P of a virtual reference point coupled with the three axles in the way providing uniform distribution of forces between the axles. Both approaches give the same loading effect but the second one enables better control of the process allowing undoubtful reaching of the ultimate load being determined as the highest force value of the force-displacement relationships.

The vehicle load position near the quarter of the span has been defined as the most unfavourable one for the arch identified in a previous work on the analysis of the bridge under moving loads [4]. The live loading was applied considering incremental level of intensity until failure of the arch by a hinge mechanism, determining the ultimate load capacity of the bridge models.

3. COMPARISON OF RESULTS

The comparison of the results (given in Tab. 4) obtained by means of all models is based on three phases of the structure response to the applied loads: 1) equilibrium under the dead load, 2) additional action of the vehicle loading *P* with intensity corresponding to the standard value, 3) the ultimate load capacity of the bridge reached by increase of *P*.

Table 4. *Results of analyses.*

Loads	Model	Level of *P*	d_v [mm]	w_n [mm]	σ_2 [MPa]	Hinges
	FE/DC		1,65 -	0 -	-0,53 -	
dead	FE/CT	0.0 P	1,05 (-36%)	0 (0%)	-0,44 (-17%)	-
	DE		1,67 (+1%)	0 (0%)	-0,62 (+17%)	
dead +	FE/DC		9,66 -	0,82 -	-2,39 -	
standard	FE/CT	1.0 P	5,70 (-41%)	0,23 (-72%)	-1,38 (-42%)	-
live	DE		10,00 (+4%)	0,81 (-1%)	-1,98 (-17%)	
	FE/DC	4.6 P	105,1	48,1 -	-14,9 -	B, C, D
dead +	FE/CT	4.5 P	76,0 (-28%)	21,0 (-56%)	-10,0 (-33%)	B, C, D
ultimate	DE	5.1 P	102,7 (-2%)	26,6 (-45%)	-16,1 (+9%)	B, C, D
live	RB	4.6 P	248,0 (+136%)	- -	- -	A, B, C, D

The Tab. 4 includes: intensity level of *P* corresponding to the above-mentioned phases, vertical displacement d_v of the arch and normal displacement w_n of the joint at hinge B, minimum principal stress σ_2 within the arch blocks and the evolution of hinges' formation. Modes of failure generated by means of the models are presented in Fig. 5.

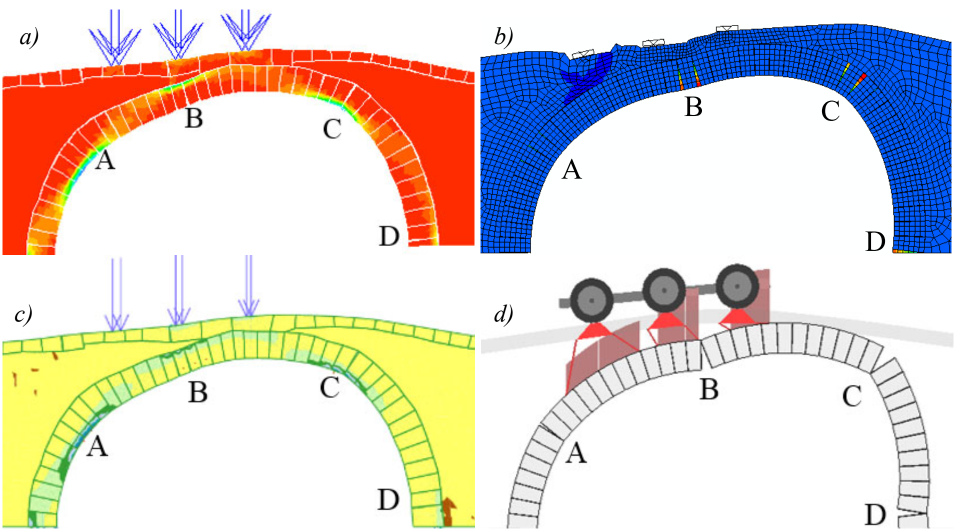

Fig. 5. *Modes of failure generated by means of: a) FE/DT, b) FE/CT, c) DE and d) RB models.*

4. CONCLUSIONS

The presented study compares four common methods applied to analysis of a masonry bridge. It confirms good agreement between the results mainly in terms of the hinges configuration and the vehicle loading intensity level at the bridge failure. A larger discrepancy is found between the values of displacements and stresses.

Assumed (however very low) tensile strength of the arch mortar joints within the FE/CT model does not seem to enlarge the flexural resistance and stiffness of the arch as the obtained stresses are even much lower for this model than for the other ones with non–tensile strength of the joints.

Significant influence of the way of live loading application on the results generated in each model is predicted. This is related also to some general differences between the models in the pavement representation.

However, despite the essential differences in formulation of the models and the procedure of the analysis a satisfactory compatibility can be obtained provided correspondence of the constitutive models.

REFERENCES

[1] COSTA C., ARÊDE A. and COSTA A., Mechanical characterization of the constituent materials of stone arch bridges, *9th International Masonry Conference*, Guimarães, 2014.

[2] COSTA C., ARÊDE A., MORAIS M. and COSTA A., Detailed FE and DE modelling of stone masonry arch bridges for the assessment of load-carrying capacity, *1st International Conference on Structural Integrity*, Funchal, Madeira, Portugal, 2015.

[3] KAMIŃSKI T., Mezomodelling of masonry arches, 6th International Conference *AMCM'2008 – Analytical Models and New Concepts in Concrete and Masonry Structures*, Łódź, 9-11 June, 2008; pp. 359-360.

[4] COSTA C., COSTA P., ARÊDE A. and COSTA A., Detailed FEM modelling of stone masonry arch bridges under road traffic moving loads, *M. Papadrakakis, M. Fragiadakis, V. Plevris (Eds.), 3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering - COMPDYN 2011*, Corfu, Greece, 2011.