

Collapse analysis of Colosseum arcades by seismic events

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ABSTRACT: On the basis of historical documentations the paper assumes the earthquake as the most probable cause of failure of the missing arcades of the Colosseum. A rigid-plastic no-tension model is adopted to perform the limit analysis in terms of upper bound approach with the earthquake treated as a load multiplier. Particular attention is paid to the identification of the three-dimensional collapse mechanism, which strongly depends on the plan of the monument. Given the direction of the earthquake, a unique collapse mode corresponding to the lowest value of the load multiplier can be identified as function of the seismic intensity.

1 INTRODUCTION

The preservation of the architectural heritage puts numerous problems of a different nature involving various disciplinary fields. From the structural point of view these problems concern the estimation of the ultimate load of the monument in its present condition on the basis of a proper mechanical model, which must also identify the most probable events that determined the actual damaged state.

Indications for the choice of the mechanical model can be derived from the analysis of the damages that can be observed in the structure. In the case of the Colosseum, the damages preserved in the restoration work performed by Stern (1814-1820) are of particular importance, since they offer a picture (Fig. 1) of what occurred in the past as a consequence of the dynamics started according to some collapse mechanism. The choice of the proper mechanical model needs therefore the comprehension of the significant events of the “structural” history of the monument.

For the Colosseum this history is particularly complex because of the effect of fires, the action of earthquakes, the despoliation of the materials used for other buildings and the geotechnical features of the foundation soil (Bozzano et al. 1995). These events offer an aleatory picture that makes it difficult to reach sure conclusions on the way of collapse of the missing parts.

In spite of this, the documents on the seismic history of the monument induce to believe that earthquakes were the most credible cause of failure of the missing parts. In fact, the catalogue of the historical earthquakes (Boschi et al. 2000) occurred in the country area surrounding Rome, and in particular those that determined serious damages in many Roman buildings, supports our assumption. Among these numerous seismic events, the earthquake of 1349 seems to be the most probable that, also for its intensity, determined the collapse of some arcades of the Colosseum, as tested by Petrarca in a letter of the same year: “*cecidit aedificiorum veterum neglecta civibus, stupenda peregrinis mole*”.¹

The date of the earthquake of 1349 also agrees with the history of the monument; used as arena until the VI century, the Colosseum became in the following centuries, especially during

¹ Roberto Luciani, *Il Colosseo*, De Agostini, Milano, 1993, p. 195.

the Middle Ages, a cave of valuable materials to be used in other buildings. In particular, the despoliation of the bronze devices connecting the various parts of bearing elements transformed the structure, initially conceived for “closed box behavior”, into a system made of elements simply supported each other and therefore susceptible of relative movements.

Thus, an investigation of the effects of seismic actions seems to be mandatory. The novelty of the analysis presented in this paper with respect to previous studies (Como et al. 2001) consists in considering that the collapse due to seismic events requires the contemporaneous activation of both circumferential and radial mechanisms of the outer wall of the Colosseum.



Figure 1: Damages preserved in the restoration work by Stern (1814-1820).

2 THE MECHANICAL MODEL

2.1 *Plan dimensions and geometrical model*

The plan of the Colosseum (Fig. 2) is characterized by an elliptical contour, which can be well approximated by a polycentric curve corresponding to four arcs of circles characterized by two different radii (Fig. 3).

The main bearing stone structures are disposed along the local radial and tangential directions of the polycentric rings. The two outer circumferential walls are made of three-level arcades whose columns are connected in the local radial direction by vaults covering the ambulatories at

the various levels. The pillars, connected by arcades in the local tangential plane, are located along the two outer polycentric rings in the same directions of the radial bearing-walls.

The fact that the radial vaults are in *opus caementicium* while the circumferential arcades are made of travertine elements allows to assume that the mechanism of the structure, whatever it can be, involves the system of arcades of one or both the two outer rings, while the radial vaults covering the ambulatories do not affect the mode of the mechanism; they must be anyway taken into account as supported loads.

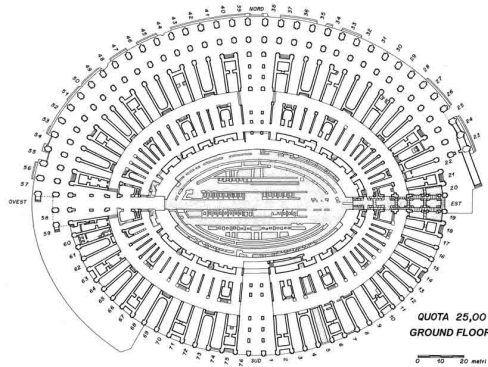


Figure 2 : The elliptical plan.

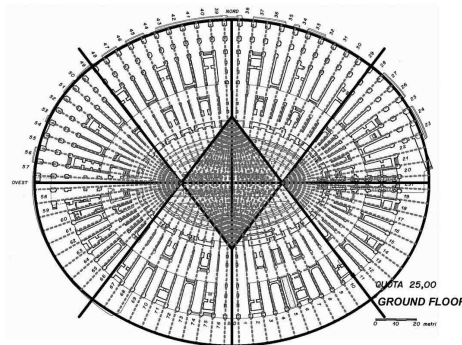


Figure 3 : Approximated polycentric curve and directions of the radial bearing-walls.

2.2 Constitutive relationship

Since the assumed main bearing structures of the monument are arcades made of travertine elements, the mechanical model is that of a structure infinitely resistant to compressive stress and characterized by no-tension behaviour at the joints connecting adjacent elements (Heyman 1982). Thus, the model is that of a rigid-plastic structure subject to friction and unilateral constraints.

2.3 The earthquake as static load multiplier

The special geometry of the structure implies that, in the presence of an earthquake treated as a static load multiplier, the seismic action on each column of the arcades depends not only on the direction of the horizontal earthquake but also on both the local curvature and the position of the column along the polycentric ring.

Let K_s be the value of the seismic acceleration in g units, treated as a static acceleration, and assume that the earthquake acts along the local radial direction corresponding to the 45-th column, as shown in Figs. 4-5; this seismic direction is in accordance with the possible ones of the multiple epicentres of the 1349 earthquake.

The radial and tangential seismic actions on each column are shown in Fig. 4, while in Fig. 5 the resultant actions can be observed; notwithstanding the interactions due to the reciprocal thrusts between adjacent arcades could suggest that the opening of this annular structure is quite impossible, the resultant actions on the columns (Fig. 5) indicate a probable radial outward collapse mode of half of the structure accompanied by the corresponding tangential mechanism.

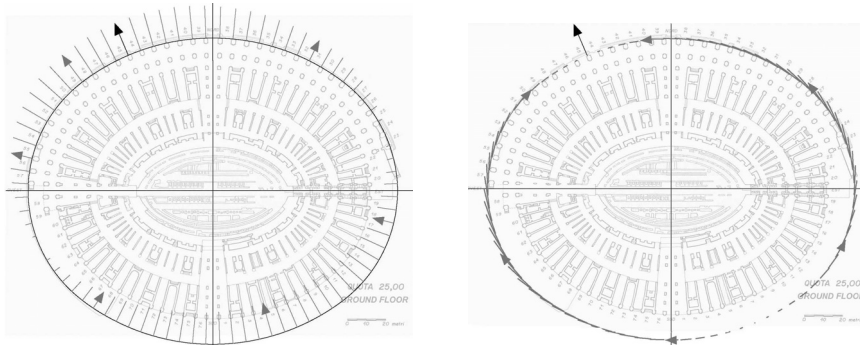


Figure 4 : Radial and tangential components of the earthquake (resultant seismic vector in black).

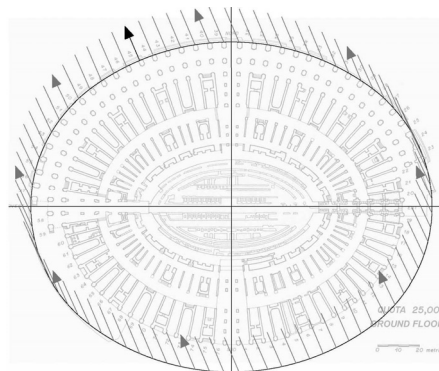


Figure 5 : The resultant acceleration of the earthquake acting on the columns.

2.4 Kinematics, collapse mechanism and amplification factor

The most probable global mechanism of the structure is expected to be that with movements of the columns – both in the local radial and tangential directions - in phase with the seismic actions; it is a three-dimensional mechanism involving contemporaneously the local tangential mechanism of the three-level arcades and the local radial collapse of the columns. To determine the global mechanism, let us assume that the elliptical contour is approximated with a polygonal line and that each three-level arcade lies on the vertical plane defined by two adjacent columns; thus, the elliptical geometry implies as many local vertical planes as the number of arcades. In these planes, the mechanism is the local tangential mechanism, while the radial mechanism lies in the local radial vertical plane defined by each column.

In the global mechanism suggested by the resultant seismic actions of Fig. 5, about half of the outer ring remains at rest since the inward movements are not allowed by the radial walls. In the global mechanism two arcades, located nearly symmetrically with respect to the seismic direction, can be identified which are characterized only by tangential displacements. By starting

from these arcades and symmetrically approaching the column whose local radial direction coincides with that of the earthquake, the local tangential displacements of the columns decrease, while the radial components increase more and more approaching the earthquake direction, until the displacement becomes only radial.

Any global mechanism in phase with the seismic action involves a large number of columns, ranging from an unknown lowest value to the maximum of 40 columns. The necessity to activate both radial and tangential mechanisms determines the interplay between seismic intensity and number of columns involved. The necessary condition to activate a possible global mechanism requires a local radial seismic action greater than the “seismic threshold” $K_s^{c,r}$ of the radial collapse of the column considered as isolated. This means that the seismic acceleration characterized by a given intensity $K_s > K_s^{c,r}$ and a given direction decides the number of arcades involved in the activated global mechanism.

The sufficient condition for the kinematical admissibility of the global mechanism requires zero value of the virtual work performed by all the forces acting on the structure; the collapse mechanism corresponding to the lowest load multiplier can be thus identified.

To identify the local tangential mechanism, let us assume that the first two three-level starting arcades belong to the same tangential plane; the orientation of the tangential movement is from right to left, as shown in Fig. 6.

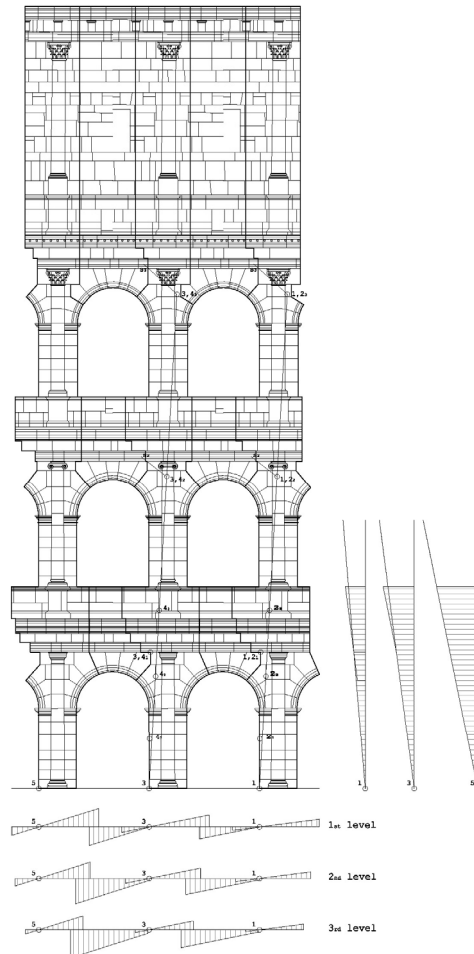


Figure 6 : Plane mechanism of the three-levels arcades and amplification of the rotation angle.

Necessary considerations of a dynamical character concerning the fixed position of the rotation axis at the ground edge of each column could suggest to identify the centre of relative rotation between the right column and the corresponding adjacent part of the three-level arcade at the centre of oscillation of the column, which lies on the diagonal plane crossing both the ground edge and the centre of mass of the column. However, the column is in contact with three adjacent key voussoirs (each one formed by a certain number of blocks) at the different levels making undetermined the identification of the corresponding centres of relative rotation.

It is important to outline that the stone elements above the extrados of the key voussoirs of the arch must have a structural function in order to respect the impenetrability law; thus, they cannot be considered as simply supported loads. As a consequence, all the rotational mechanisms without sliding are not allowed. We assume to identify the sliding-rotational mechanism characterized by a non-associative flow rule by observing the actual relative sliding well evident at all the three levels in Fig. 1 and by imposing conditions of kinematical compatibility; they require that both the relative sliding and the size of the “extended key voussoir” increase with the level. Thus, the centres of relative rotation at the different levels have been chosen according to the schemes shown in Fig. 6 and Fig. 7.

The asymmetric key voussoir moves by roto-translation with respect to the right column and by sliding with respect to the left column (Fig. 7); the tangential mechanism for the upper level arcades can be consequently determined.

The tangential mechanism of the first two starting arcades, considered as belonging to the same plane, would imply a “domino effect” which is actually reduced in the three-dimensional kinematical chain defining the global mechanism; the radial and tangential kinematical chains can be obtained by taking into account the plan geometry and propagating along the polygonal contour the kinematical chain determined in the local tangent plane.

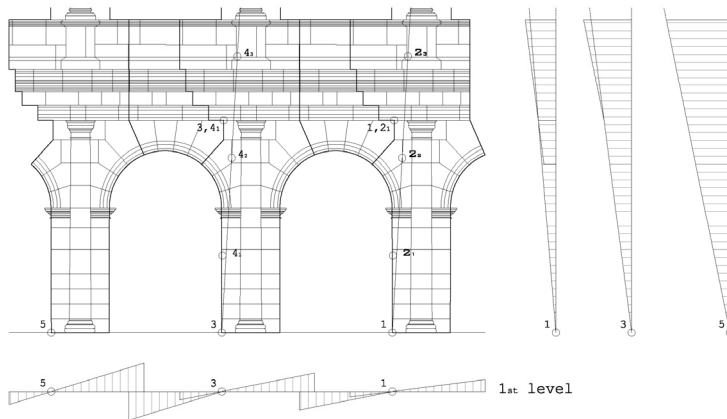


Figure 7: Details of the mechanism of the ground arcade.

2.5 Limit analysis: Active, resistant and dissipative work

The collapse condition of the structure in accordance with a given mechanism requires that the virtual work performed by all the forces acting on the system be null; we include in our analysis the dissipation work due to friction along the joints where relative sliding are allowed. Remind that the necessary condition to activate the global mechanism requires a seismic intensity K_s in g units greater than the corresponding radial collapse acceleration of the column, so that the intensity of the earthquake decides the number n of arcades involved in the mechanism.

In order to evaluate the work for each arcade, consider that the propagation of the movements from a given arcade to the subsequent one lying on a different tangential plane requires the

evaluation of both tangential and radial components of the displacement; with this aim, let us associate to the i -th arcade only one column and the adjacent key voussoirs at the three levels.

The virtual work of the seismic forces $\delta L^{(a)}$ is performed along both radial and tangential displacements; if j denotes the level of the i -th arcade, the expression of the virtual work due to the earthquake is:

$$\delta L^{(a)} = \sum_{i=1}^n g \left[m_i^c \left(K_s^t \delta r_i^{c,t} + K_s^r \delta r_i^{c,r} \right) + \sum_{j=1}^3 m_i^v \left(K_s^t \delta r_{j,i}^{v,t} + K_s^r \delta r_{j,i}^{v,r} \right) \right] \quad (1)$$

where gK_s^r and gK_s^t = radial and tangential seismic acceleration, m_i^c and m_i^v = column and key voussoir mass of the i -th arcade, $\delta r_i^{c,r}$ and $\delta r_i^{c,t}$ = radial and tangential virtual displacement of the column of i -th arcade, $\delta r_{j,i}^{v,r}$ and $\delta r_{j,i}^{v,t}$ = radial and tangential virtual displacement of the key voussoir of j -th level and i -th arcade.

The virtual work $\delta L^{(w)}$ performed by the weight of each element is:

$$\delta L^{(w)} = \sum_{i=1}^n g \left[m_i^c \left(\delta r_i^{c,t} + \delta r_i^{c,r} \right) + \sum_{j=1}^3 m_i^v \left(\delta r_{j,i}^{v,t} + \delta r_{j,i}^{v,r} \right) \right] \quad (2)$$

where g = gravity acceleration, and the meaning of the other symbols used in Eq. (2) is the same as in Eq. (1).

The dissipative virtual work $\delta L^{(d)}$ is due to the friction performance at both the right and left joints of the key voussoir for each arcade at each level. The evaluation of this work requires that the value of the thrust acting on the arch be determined; this can be easily made by considering that the resultant reaction is along the Coulomb cone both at the left and right joint of the asymmetric key voussoir. Therefore, the expression of the dissipative work becomes:

$$\delta L^{(d)} = -\mu \sum_{j=1}^3 \sum_{i=1}^n \left[\left| R_{j,i}^{v,n} \right|_{left} \left| \delta r_i^{c,t} - \delta r_{j,i}^{v,t} \right|_{left} + \left| R_{j,i}^{v,n} \right|_{right} \left| \delta r_i^{c,t} - \delta r_{j,i}^{v,t} \right|_{right} \right] \quad (3)$$

where μ = friction coefficient, $R_{j,i}^{v,n}$ = normal reaction evaluated at the left and the right joint respectively of the key voussoir at j -th level of i -th arcade, $\left| \delta r_i^{c,t} - \delta r_{j,i}^{v,t} \right|$ = relative displacement of the key voussoir at j -th level of i -th arcade with respect to the column, displacement now evaluated along the local direction of the left and the right joints.

In conclusion, the collapse condition of the structure requires that:

$$\delta L^{(tot)} = \delta L^{(a)} + \delta L^{(w)} + \delta L^{(d)} = 0 \quad (4)$$

corresponding to the search for the minimum value of the seismic multiplier $K_s > K_s^{c,r}$ satisfying (4) to which a unique mechanism involving n arcades is associated.

3 RESULTS AND DISCUSSIONS

As it is expected, given the direction of the earthquake, we have found a unique value for the lowest collapse multiplier, which depends on the value of the friction coefficient μ . For $\mu=0.6$ the lowest value of the collapse multiplier is: $K_s=0.1$, in g units, for the collapse mechanism which includes the participation of the attic, and $K_s=0.16$, in g units, without the participation of the attic; in both the cases, the number of collapsed columns is 32.

By varying the seismic direction, the total number of columns involved in the collapse does not change, but varies the number of collapsed columns at the left and right side with respect to the seismic direction.

Since the intensity of the earthquake cannot be scheduled, we have considered also values of the load multiplier greater than the lowest collapse multiplier; this circumstance takes also into account possible amplification effects of the earthquake due to the geotechnical features of the foundation soil. By increasing the value of K_s , the number of arcades involved in the collapse increases and the corresponding virtual work obviously becomes positive. The maximum number of collapsing columns is 39, a value closer to the actual state of the Colosseum; for $\mu=0.6$, the corresponding lowest value of the load multiplier is: $K_s=0.4$, in g units.

It can be interesting to observe that the assumed mechanism implies a displacement wholly lying in the vertical plane of the first starting arcade; this circumstance does not allow the activation of the radial collapse. This is exactly what can be observed in Fig. 1, where also the damages due to the relative sliding at the contact joints of the key voussoirs are shown; observe that in Fig.1 the mechanism refers to the left arcade, which is the first to move, while in the right arcade only the key voussoir has moved vertically since the right column has remained at rest. Details of sliding occurred at the third level are shown in Fig. 8.



Figure 8: Detail of the key voussoir of the third level arcade.
Left: three-dimensional simulated mechanism. Right: actual damaged state.

4 CONCLUSIONS

In this paper the collapse of the missing parts of the Colosseum has been interpreted as a consequence of the earthquake treated as collapse load multiplier. Depending on the features of the seismic actions different typologies of failure mechanisms have been identified as far as the number of the involved arcades and their distribution with respect to the seismic direction. The assumed mechanism is compatible with the numerous kinematical restrictions imposed by the complex geometry of the monument and agrees with the damages of the structure.

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