The phenomenon of veering in the assessment of r.c. arch bridges

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ABSTRACT: In this paper, possible sources of eigenvalue curve veering in arch bridges are sought. First, curve veering for an existing r.c. arch bridge is shown to be the effect of either a change in the elastic modulus of concrete, or the presence of zones with different stiffness. Then, the theoretical analysis of this phenomenon is carried out for an arch where a small region exists with elastic properties different from the remaining structure. The former example is meant to show that real arch bridges can be prone to the eigenvalue curve veering; hence, the experimental observation of veering should effectively address the identification of the structural parameters. In addition, the latter example suggests that, in some instances, localized damage in an existing bridge can be identified through the occurrence of eigenvalue curve veering.

1 INTRODUCTION

A great number of r.c. arch bridges were erected in Italy in the first half of the 20^{th} century, as the concrete arch was the most common solution adopted for bridges spanning over 40.0 m or more. Of course, many of these bridges, which were designed according to what today are considered outdated code regulations, are still in service, and their assessment is of increasing concern. In addition, current bridge inspection techniques, based on visual inspection conducted by experienced engineers, may be sometimes not suitable, or easy to apply, to arch bridges, especially when the bridge spans over a deep valley or a river.

The possibility of identifying (assessing) damage in existing structures through dynamic measurements is well known for a long time: see, e.g., the review of papers on this topic surveyed in (Doebling et al., 1998). Recently, Cerri and Vestroni (2003) developed identification techniques to detect one or multiple cracks in vibrating beams through frequency measurements. A similar approach was employed by Lepidi et al. (2005) for damaged suspended cables, as well as by Cerri and Ruta (2004) for circular arches.

In this paper a different possibility of detecting localized damage in arch bridges is investigated, based on the analysis of the eigenvalue curve. Indeed, when plotted versus any mechanical characteristics of the arch, two adjacent frequency loci may either intersect or repel each other: these two behaviours are known as “eigenvalue curve crossing” or “veering”, respectively.

The occurrence of eigenvalue curve veering and mode localization, as a consequence of the presence of a weak link between structural elements of comparable stiffness, was pointed out, e.g., by Liu (2002); he suggested that suitable derivatives of the eigenvalues and eigenvectors of the dynamic system can be used as indicators of this phenomenon. A more general discussion on the veering of the frequency loci as a consequence of perturbations in the dynamic system can be found in (Chen and Kareem, 2003). These perturbations can be either of geometrical nature (e.g, lack of symmetry or springer misalignment in arch bridges), or originate from inhomogeneities in the mechanical properties, as in the case of damage localized at any position in a symmetric arch.

The aim of the present work is twofold. First, the occurrence of eigenvalue curve veering on existing r.c. arch bridges will be pointed out (Sec. 2). Then, the possibility of detecting any localized structural damage in arch bridges will be theoretically investigated through the study of the free vibration of circular arches embedding a weaker link (Sec. 3), for which either crossing or veering in the eigenvalue curve occurs according to the position of the damaged zone.

2 THE PHENOMENON OF VEERING IN EXISTING R.C. BRIDGES

The crossing and veering phenomena should be frequently encountered in the dynamics of arch structures, as in the literature these characteristics are often related to curved structures for which some classes of eigenvalues and eigenvectors show a strong dependence on a geometric or mechanical parameter. For example, according to the 2D F.E. modelling of the r.c. arch bridge shown in Fig. 1, about 90 m long and currently investigated by the authors, the following comments can be made:

a) if a perfectly symmetric model is adopted, the eigenvalue loci of the first two flexural modes (1^{st} symmetric and 1^{st} anti-symmetric) exhibit a crossing behaviour *versus* the (normalized) Young's modulus, E, of the structure (that is, *versus* the stiffness-to-mass ratio), as shown in Fig. 2a. In this figure, E_0 is a reference value for the elastic modulus;
b) if a slight non-symmetry is introduced in the model (2% longitudinal slope), the eigenvalue loci of the same modes (again plotted versus the Young's modulus), after approaching, veer instead of crossing each other (Fig. 2b). Unlike the case of crossing, an important characteristic of the veering of the eigenvalue loci is that the mode shapes associated with each locus are combined during the veering in a continuous way. This originates so-called hybrid modes, strongly evident in the approaching zones (Fig. 2b);
c) if the non-symmetry is introduced by a change in the Young's modulus in a small portion of the structure, as shown in Fig. 3a, again the veering phenomenon arises (Fig. 3b).

Furthermore, keeping in mind that only a small number of full-scale dynamic tests have been carried out on modern r.c. arch bridges (Cantieni et al. 1994, Benedettini et al. 2005, Gentile 2006), it has to be mentioned that Benedettini et al. (2005) reported that two arch bridges in the Teramo Province (central Italy) exhibited an anomalous dynamical behaviour, that can be traced back to the veering of two neighbouring vibration modes. In the case of *Valle Castellana bridge*, the sequence of the modes hybridization is related to the first two flexural vibration modes, whereas in the case of *Frattoli bridge* the hybridization sequence pertains to the 1^{st} anti-symmetric flexural mode and the 1^{st} torsion mode.

Figure 1 : View of the tied-arch bridge in Canonica d'Adda, Italy (1955).

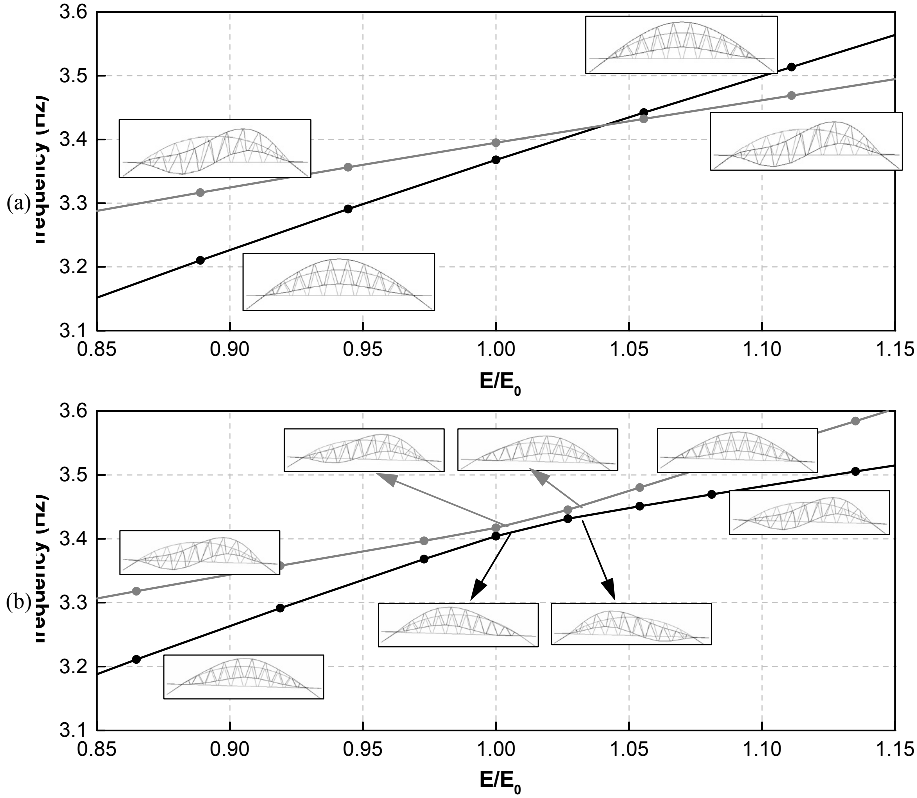

Figure 2 : (a) Crossing on a symmetric and (b) Veering on a non-symmetric arch bridge model.

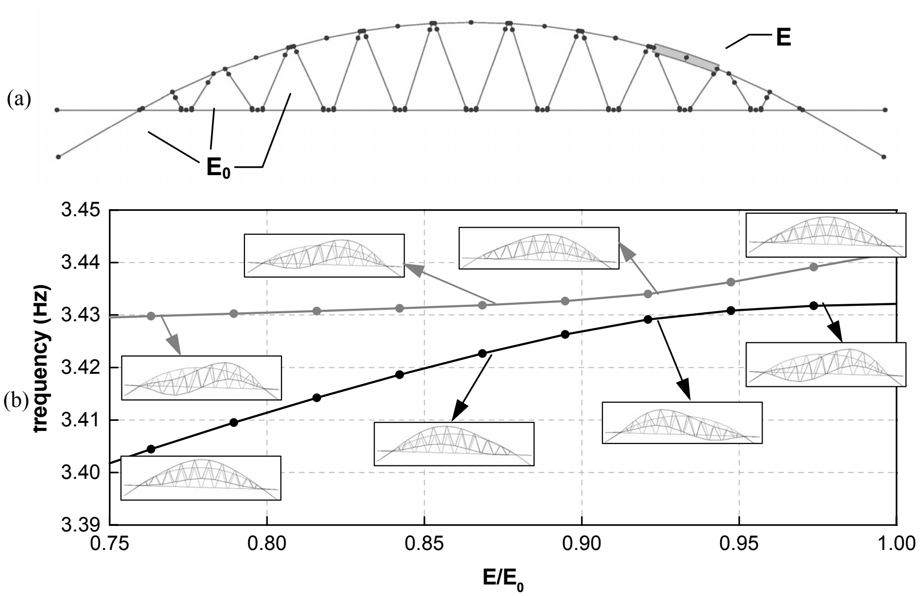

Figure 3 : (a) Damaged arch bridge model; (b) Veering on the damaged arch bridge model.

3 THE PHENOMENON OF VEERING IN DAMAGED ARCHES

3.1 Problem position

Consider a double-hinged circular arch with a symmetry axis y; the cross-section of the arch is constant and symmetric respect to the (x,y) plane. Let R denote the arch radius, $2\varphi_0$ the angle of opening of the arch, J the cross-sectional moment of inertia, μ the weight per unit arch length. The arch is homogeneous, except for an arbitrarily small neighbourhood of any section where damage is supposed to be localised: let E be the Young's modulus of the homogeneous (undamaged) material. According, e.g., to Cerri and Ruta (2004), the damaged section can be modelled as a spring of stiffness k lower than the remaining arch (see Fig. 4): the angle φ_{cr} gives the position of the damaged section, measured clockwise from the symmetry axis y.

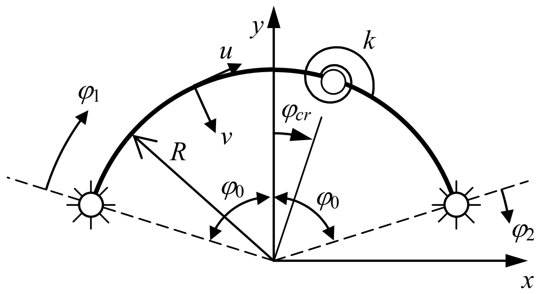

Figure 4 : Symmetric circular arch embedding a damaged section.

It is expedient to use two angle coordinates, φ_1 and φ_2, running clockwise along each of the two parts of the arch and vanishing in either one of the hinges (Fig. 4). Let u and v denote the axial and transversal displacements of the centre line of the arch, respectively: assuming the arch to be thin, shear strains can be neglected and the rotation θ of any cross section is $\theta = (u+dv/d\varphi)/R$. Also, provided that the arch is not shallow, it is reasonable to assume that the centre line of the arch is incompressible, so that $v = du/d\varphi$. According, e.g., to Henrych (1981), neglecting the effect of shear, rotary inertia, and tangential inertia forces, the equation of the free vibration of an incompressible arch is

$$\frac{d^6u}{d\varphi^6}+2\frac{d^4u}{d\varphi^4}+(1-\chi^2)\frac{d^2u}{d\varphi^2}=0, \tag{1}$$

where $\chi^2 = \mu R^4\omega^2/(EJ)$, ω being the circular frequency; only positive values of χ will be taken into account. Setting $\eta = \sqrt{(\chi+1)}$, $\kappa = \sqrt{(\chi-1)}$, and disregarding the particular case where $\chi=1$, the general solution of Eq. (1) reads

$$u_i(\varphi_i) = A_i\sin(\eta\varphi_i) + A_{i+1}\cos(\eta\varphi_i) + A_{i+2}\sinh(\kappa\varphi_i) + A_{i+3}\cosh(\kappa\varphi_i) + A_{i+4}\varphi_i + A_{i+5}, \tag{2}$$

with $i = 1$ for $0 \le \varphi_1 \le \varphi_0+\varphi_{cr}$ and $i = 2$ for $-\varphi_0+\varphi_{cr} \le \varphi_2 \le 0$. The set of boundary conditions for the arch is equivalent to a homogeneous system of algebraic equations of the form $[S(\chi,\varphi_{cr})]\{A\}=\{0\}$, with $\{A\} = \{A_1, A_2 \dots A_{12}\}^T$. Non-trivial solutions are obtained if $[S]$ is singular, i.e., if

$$\det[S] \equiv kg_1(\chi) + g_2(\chi,\varphi_{cr}) = 0, \tag{3}$$

with

$$g_1(\chi)=((2\eta\kappa)^2\chi)^3\sin(\eta\varphi_0)\sinh(\kappa\varphi_0)\times$$
$$\times[2\eta\kappa\chi\varphi_0\cos(\eta\varphi_0)\cosh(\kappa\varphi_0)-\kappa^3\sin(\eta\varphi_0)\cosh(\kappa\varphi_0)-\eta^3\cos(\eta\varphi_0)\sinh(\kappa\varphi_0)], \tag{4a}$$

and

$$g_2(\chi,\varphi_{cr})=-4\chi^4(\eta\kappa)^5\times\{\kappa\eta^3[\cos(2\eta\varphi_0)-\cos(2\eta\varphi_{cr})]+$$
$$+2\kappa\eta\cosh(2\kappa\varphi_{cr})[\kappa^2\sin^2(\eta\varphi_0)-\eta\chi\varphi_0\sin(2\eta\varphi_0)]+$$

$$+\kappa\cosh(2\kappa\varphi_0)[\eta(\eta^2\cos(2\eta\varphi_{cr})-2\eta\cos(2\eta\varphi_0)-\kappa^2)+2\chi\eta^2\varphi_0\sin(2\eta\varphi_0)]+$$
$$+2\chi\kappa^2\eta\varphi_0\sinh(2\kappa\varphi_0)[\cos(2\eta\varphi_0)-\cos(2\eta\varphi_{cr})]+$$
$$+4\sin(\eta\varphi_0)[2\eta^2\cosh(\kappa\varphi_{cr})\sinh(\kappa\varphi_0)(2\cos(\eta\varphi_0)+\kappa^2\cos(\eta\varphi_{cr}))+$$
$$-\sinh(2\kappa\varphi_0)((4+\eta^2\kappa^2)\cos(\eta\varphi_0)+2\kappa^2\cos(\eta\varphi_{cr}))]\} \tag{4b}$$

Note that the eigenfrequencies of the perfect (undamaged) arch correspond to the roots of $g_1(\chi) = 0$: these frequencies may be compared with those measured on any real arch to detect the presence of damage. The first eigenfrequencies of perfect circular arches of different geometry are listed in tabular form in Henrych (1981). Also note that $\chi = 0$ or 1 are roots of Eq. (3) irrespective of φ_{cr}: the first root is clearly immaterial; the other one is not acceptable, as Eq. (1) degenerates if $\chi = 1$ and Eq. (2) is no longer its solution.

In the following section the numerical solution of Eq. (3) is sought. In particular, the influence of the position of the damaged section on the free vibration of the arch is analyzed.

3.2 Influence of the position of the damaged section φ_{cr} on the free vibration

Consider first the case of a symmetrically damaged arch ($\varphi_{cr} = 0$). In this case Eq. (4b) can be rewritten as

$$g_2(\chi)=-8\chi^4(\eta\kappa)^5\sin(\eta\varphi_0)\sinh(\kappa\varphi_0)\times\{-\kappa\eta\sinh(\kappa\varphi_0)[\sin(\eta\varphi_0)+\varphi_0\chi\eta\cos(\eta\varphi_0)]$$
$$+4[\eta^2(\kappa^2+2\cos(\eta\varphi_0))-\cosh(\kappa\varphi_0)[(4+\eta^2\kappa^2)\cos(\eta\varphi_0)+\eta^2(2+\varphi_0\chi\eta\sin(\varphi_0\eta))]]\}\,. \tag{5}$$

This shows that a set of eigenfrequencies corresponds to $\sin(\eta\varphi_0) = 0$, that is

$$\chi=\left(\frac{n\pi}{\varphi_0}\right)^2-1\,,\ n = 1, 2, \ldots \tag{6}$$

The corresponding eigenmodes are anti-symmetric: these modes are insensitive to the presence of the damaged section as the curvature of the vibrating centreline vanishes at the crown of the arch. The remaining roots of Eq. (3) correspond to symmetric eigenmodes.

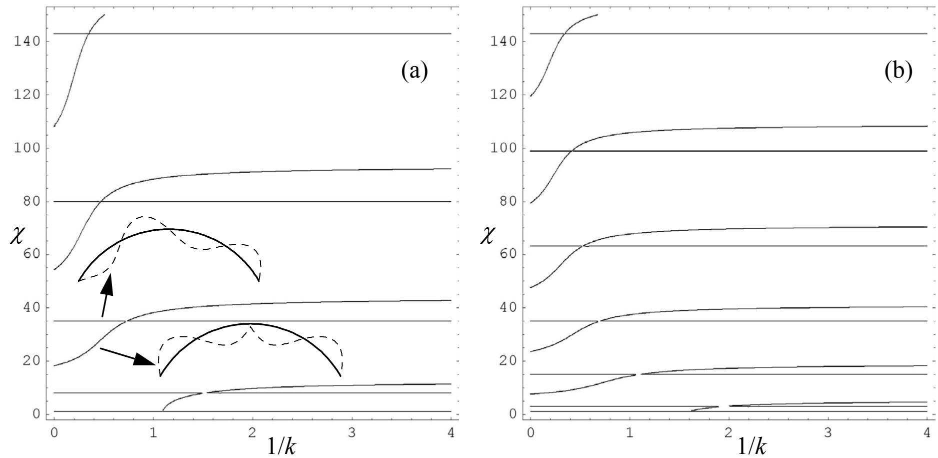

Figure 5 : Normalized eigenfrequencies vs flexibility of the damaged section for a symmetrically damaged arch. (a) $\varphi_0 = \pi/3$; (b) $\varphi_0 = \pi/2$.

In Fig. 5 the normalized eigenfrequencies (χ) are plotted versus the flexibility of the damaged section ($1/k$) for a circular arch damaged at the crown, with an opening angle $2\varphi_0$ of (a) $2\pi/3$ or (b) π. The corresponding qualitative eigenmodes are also shown. The plots show that crossing of the pairs of eigenvalue loci corresponding to the same wavenumber occurs. The case of an undamaged arch is recovered in the limit case $1/k \to 0$, whereas $1/k \to \infty$ represents an arch which has completely lost its bending stiffness at the crown.

Assume now that the damaged section has an arbitrary location ($\varphi_{cr} \neq 0$). All the roots of Eq. (3) have now to be sought numerically. Fig. 6 shows the plots of the normalized eigenfrequencies of an arch with opening angle $2\varphi_0 = 2\pi/3$ considering different locations for the damaged section ($\varphi_{cr} = \pi/72$, $\pi/24$ or $\pi/8$). The occurrence of eigenvalue curve veering is particularly evident if the damaged section is close to the arch crown, whereas the phenomenon is less clear as the damaged section approaches the haunches. As shown by the eigenmodes sketched in Fig. 6a, as the flexibility of the damaged section increases there is an exchange in the order of the eigenfrequencies that correspond to nearly symmetric and nearly antisymmetric modes of vibration.

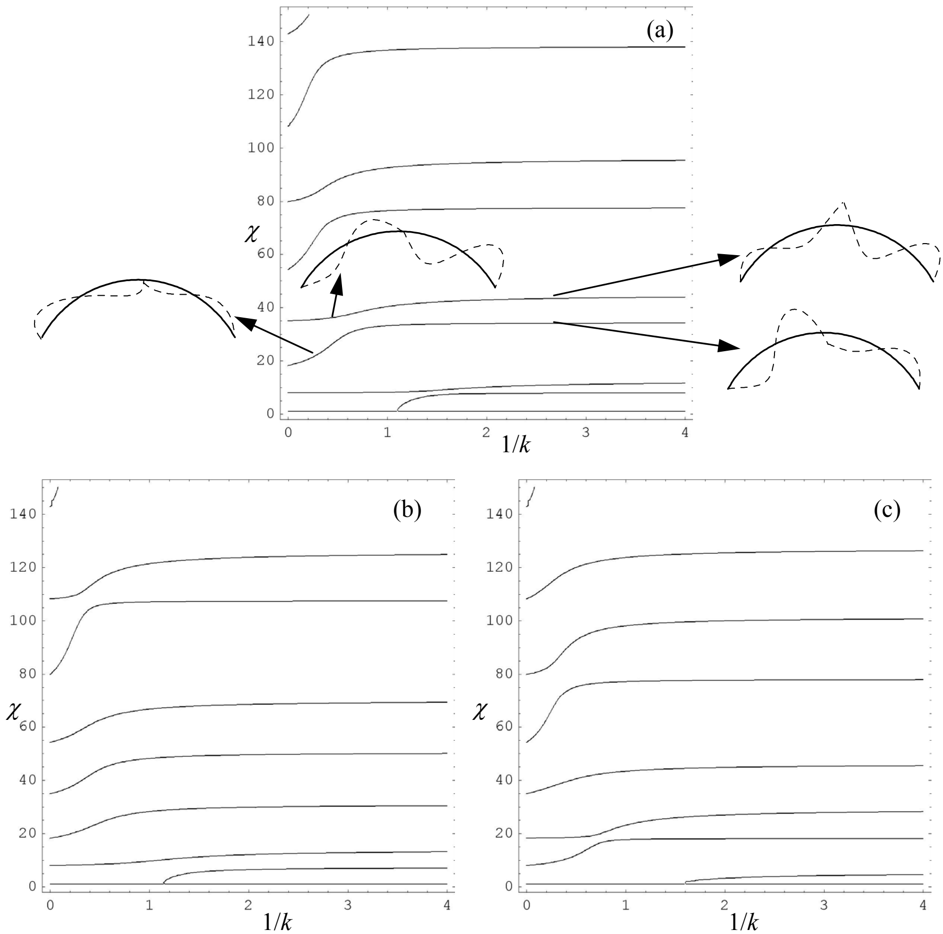

Figure 6 : Normalized eigenfrequencies vs flexibility of the damaged section for an unsymmetrically damaged arch with $\varphi_0 = \pi/3$. (a) $\varphi_{cr} = \pi/72$; (b) $\varphi_{cr} = \pi/24$; (c) $\varphi_{cr} = \pi/8$.

4 CONCLUSIONS AND FUTURE PERSPECTIVES

According to the numerical examples shown in Sec. 2 and the analytical results derived in Sec. 3, the occurrence of the phenomenon of veering in the eigenvalue curves associated to adjacent eigenfrequencies has been pointed out for arch bridges with geometrical or material imperfections. This phenomenon could be expediently exploited in identification procedures, to detect the occurrence and the intensity of any damaging process localized in an arch. In a geometrically symmetric arch, the detection of hybrid modes, the existence of close or multiple eigenvalues, or an inversion in the order of the expected modal shapes, are all symptoms of the presence of damage. Localized damage, however, might be hard to detect below a certain critical level according to the first modal shape only; indeed, according to Figs. 5 and 6, the first modal shape for a slightly damaged arch is (exactly or nearly) anti-symmetric, as in the perfect case, and the first eigenfrequency is not much different from that of the undamaged arch.

The above remarks are meant to be essentially qualitative regarding the possibility of detecting the presence of damage. The quantification of the damaging process, and the identification of the location of the damaged section(s) of the arch, require an identification procedure such as that proposed by Cerri and Ruta (2004), which makes use of an objective function defined as the norm of the difference of the predicted and the experimentally measured eigenfrequencies. The damage parameters are identified through a minimization procedure.

In the continuation of the research, the introduction is planned of the identification procedure outlined above in a FE program for 2D and 3D analyses. Also, the case of multiple damaged sections will be dealt with, and the case where damage is not localized, but rather spread over a finite portion of the arch, will be investigated.

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