A new masonry arch bridge assessment strategy (SMART)

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ABSTRACT: Although current masonry arch bridge assessment methods are able to predict the ultimate carrying capacity of the bridge with some confidence, serious concern was identified with respect to predicting long-term behaviour and residual life. A new assessment procedure (SMART) is presented which differs philosophically from all the current techniques in as much as it takes a holistic approach in considering all possible modes of failures the structure may experience under any given loading regime. Limit states are discussed and a new permissible limit state specific for masonry is proposed. From that the method enables the permissible working loads, long-term behaviour and residual life of the bridge to be found. This can be used to prioritise conflicting maintenance demands on limited budgets. The method is based upon recent research related to the long-term fatigue performance of masonry arch bridges subjected to cyclic loading.

1 INTRODUCTION

There are probably approaching a million masonry arch bridges worldwide. All are ageing and most are carrying loads well in excess of those envisaged by their builders. A recent survey (Sustainable Bridge Project) revealed that approximately 40% of the European Railway bridge stock comprises masonry arch bridges and that over 60% of these are over 100 years old. The maintenance and assessment of these bridges is a constant concern for the bridge owners.

Throughout Europe the current assessment methods fall broadly into three categories: the semi-empirical MEXE method (including a number of modified versions); elastic analysis methods with limits set on the stress levels; and ultimate limit state methods based upon a ‘mechanism’ approach or a non-linear FE analysis.

A new approach to the assessment of masonry arch bridges is presented that not only gives a more realistic assessment of current capacity but also gives an indication of residual life and hence could be used to prioritise conflicting maintenance demands on limited budgets.

2 THE ‘SMART’ ASSESSMENT METHOD

Any structural assessment method follows a similar paradigm, the Sustainable Masonry Arch Resistance Technique (SMART) is no different. This is shown in Fig. 1, where masonry arch bridges differ from many other types of bridges is that there is little current experience of the design and construction of such structures (Recently, the Highways Agency has issued a Design Memorandum which gives guidance on the Design of Unreinforced Masonry Arch Bridges).

It is vital that any assessment method takes a holistic approach; the form of construction, materials, loading etc should all be taken into account. All too often the assessment focuses upon the barrel with lesser regard to its interaction with the other elements of the bridge – when they, themselves, may be critical. Currently, Network Rail uses a modified version of MEXE as a first step in the assessment. If this yields a capacity which is too low or the type and nature of the bridge excludes its application, alternative methods of analysis and assessment are permitted.

The 'SMART' method is based upon a more holistic approach that considers: the form of construction; material properties; Limit States; actions (i.e. current and historical loading and deformation induced stresses); analysis and modes of failure. The method gives an assessment of the bridge's working and ultimate load capacity and an insight into its residual life.

1) Geometry, construction
(Incl. foundations, backfill, etc.)

↓

2) Loading

↓

3) Materials

↓

4) Structural analysis

Possible failure modes	**Determined by**
Mechanism	Arch geometry
Ring separation	Longitudinal shear strength of inter-ring mortar joint
Crushing	Compressive strength of masonry
Sliding	Shear strength of radial mortar joint
Others	…

↓

5) Ultimate Limit states (ULS)
For all possible failure modes

↓

6) Permissible Limit states (PLS)
For all possible modes of behaviour

↓

7) Assessment of carrying capacity and life expectancy
Fatigue properties of materials defined through the S-N diagrams for different fatigue failure (e.g. compression and shear action)
(e.g. application of Miner's rule)

Figure 1 : SMART assessment procedure

2.1 Limit state

In determining the ULS (collapse load) the bridge owner is assured of the ultimate capacity but little else. Based upon field tests in the 1980's, two modes of failure were reported – the formation of a mechanism comprising hinges (4 in number for a single span bridge) and a 'snap-through' failure. These modes of failure were, to some extent, pre-determined by the nature of the loading system (a full-width 'knife edge load' applied at about the quarter span). The loading was applied monotonically through to failure.

It is at this point that it is very important to appreciate that although there is some general agreement as to the definition of the ULS as the condition at which a collapse mechanism forms in the structure or its supports, no such agreement exists for other limit states. There are three other limit states that have to be considered, namely: the serviceability, fatigue and durability limit states respectively (SLS, FLS, DLS). Although it may be fairly easy to differentiate between these limit states for metal and reinforced and prestressed concrete bridges, it is not so easy for masonry arch bridges. The SLS is usually determined against criteria of crack width, deflection, vibration etc. In the case of the masonry arch bridge it is difficult to set meaningful limits for these. Clearly, it would be unacceptable to have a rail deflection of a magnitude which could derail a train. However, rail deflection will not be solely dependent upon the arch flexibility and will be subject to the same limitations set for the entire system. The FLS is quite specific for metal and reinforced and prestressed concrete bridges, it includes failure caused by fatigue or other time dependent effects. The DLS refers to the assessment of remaining service life in the context of environmental parameters. There is a strong case to bring these three limit states together for the purposes of masonry arch bridges. Bridges owners usually want to know two things: is the bridge strong enough to carry its working loads and what is its residual life. This can be achieved by assessing the ULS and what might best be called the Permissible Limit State (PLS). The PLS may be defined as the limit at which there is a loss of structural integrity which will measurably affect the ability of the bridge to carry its working loads for the expected life of the bridge. As can be seen, this brings together the critical elements of the other limit states to give a unique assessment tool for masonry arch bridges. The process involves the calculation of the stress ranges that the bridge experiences for each of the modes of behaviour and their cumulative effects in the context of an S-N curve. The current state of knowledge means that the S-N curves for each mode of behaviour/failure will be conservative and may reduce to a permissible stress (endurance limit stress), but as experience and confidence in the method grows these will be replaced by S-N curves similar to those currently used for other materials.

2.2 Geometry and construction

The first step in the SMART assessment, as well as in any other assessment method, is to determine the form of construction and geometry of the bridge. It is very important at the outset to dispel the idea that all masonry arch bridges are of similar construction – nothing could be further to the truth. The different types of construction have evolved over centuries of trial and error and technological development. As a structural form the arch can be traced back to Mesopotamian times over 4000 years ago. Certainly, the origins of the railway bridges of the nineteenth century emerged from a medieval tradition of stone arches proportioned by experience and passed down by the master masons to successive generations. Geometrical proportion therefore determined the relationships between the span, arch barrel thickness, and the abutment and pier dimensions. Fig. 2 shows a typical arch bridge construction.

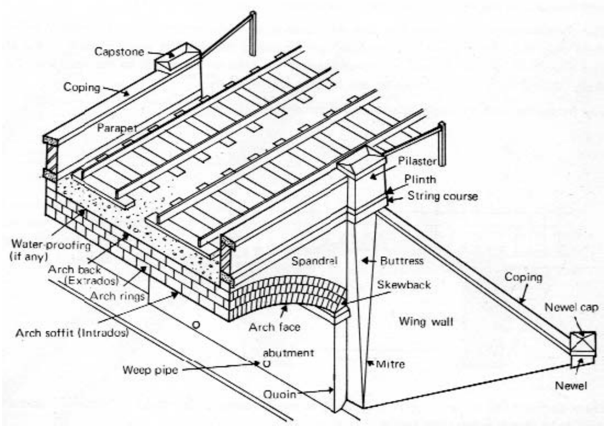

Figure 2 : Typical arch bridge construction

The barrel may take various shapes including; semi-circular, parabolic, segmental, elliptical, gothic pointed and may comprise dressed stone, random rubble, brickwork or mass concrete.

The backfill may be anything from ash and rubble through to concrete. The backfill over the arch may be contained by spandrel walls which extend beyond the abutments to provide wing walls. Clay was sometimes used as a waterproofing membrane over the arch barrel. To lighten the structure and also to eliminate the horizontal soil pressures on the external spandrel walls, internal spandrel walls were sometimes used. This form of construction was used on some bridges with spans greater than about 12 m. The proportions of these internal spandrel walls depended on the nature of the available masonry and whether or not the over-spans took the form of stone slabs or arches. Significantly, there are usually no external indications of the form of internal construction. Even the barrel thickness cannot be relied upon to correspond to that observed on the elevations as the latter was frequently proportioned on aesthetic grounds. Alternatively, internal arches may be provided which span longitudinally and spring from the extrados of the main arch barrel. These may be totally internal, or extend through the external spandrel walls to provide an aesthetic feature and, in the case of bridges over rivers, an escape route for flood water. An extension of this form of construction takes the form of a series of smaller arches supported by piers resting upon the extrados of the main span.

By the time of the coming of the railways, foundation construction had reached a level of sophistication that was well beyond the contemporaneous theoretical understanding of soil mechanics. It is a legacy to our forebears that so many of their foundations have stood the test of time. Where practicable the bridge foundations were taken down to rock. This was often not possible and so timber piles or timber grids on timber piles were used. Caissons were used for pier construction in rivers and cut-waters were used for protection against scour which was recognised as a major threat to bridge foundations. It is important to determine the geometry and form of the construction of the foundations as their load carrying capacity may be critical – especially if the loading regime is planned to be changed.

It is very important to collect information that defines the boundary conditions of the bridge. The geometrical data and construction details should therefore include the embankments etc. on the approach to the bridge as well as the dimensions of the obstacles over which the bridge carries its traffic. (If the obstacle is a river, then its geomorphology should be considered with particular reference to flood conditions and scour history).

All masonry arch bridges have some defects. Most of these may be of a minor nature and so do not affect carrying capacity. However, they cannot be dismissed and should be faithfully recorded. Routine maintenance, like re-pointing, can mask historical or even new movement of the masonry. It is important that prior to re-pointing, cracks are recorded and their cumulative effect assessed.

2.3 Materials

The second step is to consider the construction materials and their basic properties. It is beyond the scope of this paper to consider the properties of all of the materials and their combinations that might be found in masonry arch bridges. The main materials used in masonry construction include a variety of bricks and stone units, typically separated by bed and vertical joints comprising some type of mortar. In the case of dressed stone voussoir arches the interface with the mortar is conducive to good bonding and the percentage of mortar per unit volume is usually less than 2%. In multi-ring brickwork arches, this percentage rises to approximately 20% whilst the bonding becomes problematic. Additionally the brick bonding between the rings is important and if no headers are provided then ring separation is increasingly possible. This will result in tangential cracks, loss of continuum behaviour and hence reduction of the carrying capacity. Finally, a random rubble arch may have up to 40% volume of mortar with the consequential reduction in strength. The response of masonry to loads is influenced by the way in which these materials have been used in the bridge construction, their original physical characteristics and any subsequent changes, including deterioration. On this last point, it should be remembered that the majority of the masonry arch bridge stock is now in excess of one hundred years old. Guidance on the macroscopic material properties is given in current assessment 'codes', but little guidance is available if more sophisticated analyses are judged necessary.

The MEXE method deals with material properties in an empirical way by modifying the carrying capacity derived for the 'standard' case to take account of the actual condition and nature of the bridge. All other methods require the assessing engineer to make some assumptions regarding the properties of the constituent materials. These range, depending upon the method of analysis, from simplifying assumptions like infinite stiffness and strength in compression and no tensile strength to very sophisticated mathematical models which consider interface bond and non-linear behaviour of heterogeneous assemblages. What is important is that the ramifications of any limitations and/or assumptions that are made when applying the chosen approach to the problem are fully recognised. The more sophisticated FE techniques should include a parametric study, as many of the parameters which have to be defined in the mathematical model cannot be measured in the real structure. The determination of the material properties of the bridge present the assessing engineer with many problems (Edgell 2005). There has been some development in NDT methods (www.sustainablebridges.net) but detailed evaluation of insitu properties and their variation is still some way off. At present, a deterministic approach is adopted, although methodologies for probabilistic techniques are being tested as part of the European Sustainable Bridges project (www.sustainablebridges.net).

The basic properties include the elastic modulus, compressive and tensile strengths, bond strengths and shear strength. Other properties include thermal coefficient, viscous deformation, fatigue properties. Although some of these are well understood in the case of modern brickwork, the same is not true for historic brickwork and stone masonry. Consequently, a good deal of experience and judgment is needed to arrive at realistic estimates. Of equal importance are the properties of the backfill and surfacing materials and the form and condition of the foundations.

2.4 Loading

Dead loads are essential for the stability of masonry arch bridges. It is important to consider accurately the weight and distribution of the bridge and its superimposed dead loads. This may be significantly affected by the internal construction, for example where the original internal voids between longitudinal spandrel walls have been filled. The application of load factors should take into account whether or not the dead load is beneficial.

Specific guidance on the load type, magnitudes, positions, frequency, etc. may be taken from the standard appropriate for the specific bridge.

Current assessment methods use deterministic approaches to represent the load regime that the bridge experiences. More recently, probabilistic approaches have been proposed.

To date, the value and dispersal of load through the fill has been based upon equivalent static values and ‘classical’ geotechnical dispersal. This view is now being tested against a background of proposed increases in train speeds and axle loadings. Preliminary conclusions of numerical modelling of an embankment incorporating a ‘rigid’ arch opening subjected to train loadings at various speeds suggest that the horizontal pressure changes are concentrated in a zone in the vicinity of the ballast/backfill interface. Additionally, large scale laboratory tests have been undertaken to study the fundamental nature of the soil-structure interaction with granular and cohesive backfills. Initial findings have indicated the extent of the backfill that is mobilised at failure and the limited interaction at the permissible limit state (Gilbert 2006).

2.5 Structural analysis

Masonry arch bridges are extremely complex 3-dimensional structures. The range of material from which they were constructed together with the diversity of constructional detail means that great care and considerable experience is needed to develop a representational structural idealisation (McKibbins and Melbourne et al. 2006).

There are several methods of analysis currently available which range from the semi-empirical MEXE method through to the latest non-linear finite and discrete element techniques. All these methods should carry a health warning in as much as they usually focus on the structural performance of the barrel because it is considered to be the most vulnerable element of the bridge. In fact, it is just one element of the entire structural system. Consequently, it is vital that the assessing engineer takes a holistic approach to the assessment as any one of the other elements of the bridge may be the critical element.

It is important at this stage to consider ALL the modes of failure/behaviour in order to determine the load carrying capacity. Masonry arch bridges are highly redundant structures that can develop ‘releases’ in many ways by forming hinges, sliding (internally and externally), local crushing, relative ‘flexibility’, backfill failure, foundation failure etc.

An ultimate limit state (ULS) analysis must be undertaken to demonstrate that the bridge is capable of carrying its design life loading. This analysis is usually undertaken using either a mechanism type approach or an FE/Discrete Element approach. In the latter techniques, some attempt can be made to monitor the structural response as the load increases, registering when and where hinges might occur etc. It is very important to be aware that the modelling (particularly of the material properties) may preclude the formation of some modes of failure.

Apart from tensile stresses the other two main stresses that the masonry experiences are shearing and compressive stresses. The modes of failure in the case of the co-existence of the latter two stresses will depend on the relative magnitude of the shearing and normal stresses (Hendry 1991). With low normal stress, a frictional failure may occur as shown in Fig. 3a which can be represented by line (i) in Fig. 3c. Results for this type of failure can be expressed in a Coulomb type failure criterion (Eq. (1)) where τ is the shear stress at failure when normal stress σ is applied and μ is a coefficient of friction.

$$\tau = v_0 + \mu\sigma \tag{1}$$

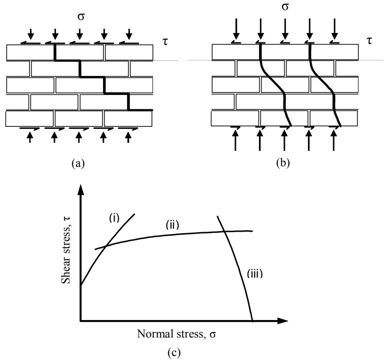

Figure 3 : Failure envelope for normal and shear stresses

Line (ii) represents the 'unit-cracking' mode of failure shown in Fig. 3b. In this case cracks pass through some head joints, but generally progress through the masonry at some angle to the bedding plane. (It is important to note that shear may not only be induced by live loading but also by relative settlement which may cause significant shearing stresses in the piers/abutments and barrel – particularly as a result of transverse settlement).

The third type of failure is straight compressive failure, represented by line (iii). This can be more complex depending upon the orientation of the principal compressive stress to the bedding plane (Page, 1991, Dhanasekar et al. 1985).

As a major addition to earlier assessments, the SMART method includes an assessment of the long-term performance of the bridge. This requires the description of the fatigue properties of the masonry. In order to quantify the fatigue performance of possible modes of failure of the masonry (i.e. crushing, tensile cracks, shear cracks which will initiate the various modes of failure of the structure), a series of S-N curves are proposed. For example, the properties of the masonry in compression can be represented by Eq. (2).

$$\sqrt{\frac{\Delta S \times S\max}{Sult^2}} = A - B\log N \tag{2}$$

Where A and B are empirically determined constants, N is the number of cycles of loading that develop a change in stress of Δs which experiences a maximum stress S_{max} compared to the ultimate strength of S_{ult} (Roberts et al. 2006)

Similar expressions are needed for each of the modes of failure.

In the SMART assessment, it is suggested that at worst the bridge will have developed sufficient releases to have reached a statically determinate state –this allows straightforward computation of the forces within the structural elements at working loads. In so doing, the influence of the internal structure and the nature of the backfill can be taken into account when determining the 'effective' active elements of the bridge and thus the range of stresses that they experience. The loading regime should then consider the distribution and frequency of the traffic, grouping it to allow analysis of the number of stress events that occur in each prescribed stress range. This will allow an assessment of residual life (using for example Miner's Rule).

2.6 Assessment and life expectancy

Currently it is assumed that the 'safe' capacity for masonry is around 50% of the ultimate load carrying capacity. This value should be compared to the PLS which is determined using the S-N curves (or the permissible stresses if the S-N curve is not available) to ensure that no accumulative damage is likely to occur below the 50% mark.

It is also important to realise that the different modes of behaviour (and their associated induced levels of stress) are not mutually exclusive i.e. the load that induced punching shear is the same load that will be inducing simultaneously longitudinal shear, flexure, tension, compression etc.

The number of cycles the structure can experience prior to any mode of failure may be investigated using Eq. (2). This requires realistic values for the stress ranges and the maximum stresses to be determined for the arch under each range of loading. This can help to indicate the effect of any change in the loading regime on the life expectancy of the bridge. For example, if the slope of the S-N curve for the compressive strength of brickwork is only 0.05, a change in the stress range parameter may have a large effect on the number of cycles to failure.

It can therefore be seen that if the range in the stress level increases from, for example, 0.25 S_{ult} to 0.5 S_{ult} and as a consequence the S_{max} increases to 0.75 S_{ult} then the maximum number of cycles to failure reduces (if A = 0.7 and B = 0.05) reduces from approximately 10^7 to $10^{1.75}$ with the consequential reduction in residual life. This is particularly significant when the line traffic regime is changed, for example, by increasing the number and axle loading of freight trains.

Subsequently the number of cycles to failure under the range of stress levels should be compared with the number of cycles the structure has experienced with the help of Miner's Rule (see Eq. (3)) where n_1 etc are the actual number of events in each designated stress range and N_1 etc are the number of such events at the corresponding stress range that would cause failure.

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots < 1 \tag{3}$$

The SMART assessment method can therefore give an estimate of residual life of the bridge and thus enable a more informed management of the bridge stock.

3 EXAMPLES

To demonstrate the 'SMART' method of assessment a simple example based upon a large-scale laboratory experiment will be used. Although this may not be representative of the 'real' problem that bridge assessment engineers face, it does demonstrate some important features of the method.

The first stage is to define the arch dimensions and constructional details. These are presented in Table 1. The 5m span arch comprises 3 rings of Engineering Class 'A' bricks laid using 1:2:9 (cement: lime: sand) mortar without headers. The abutments were attached to the structural strong floor and may be assumed to be fixed.

Table 1 : Arch dimensions

Span (mm)	5000
Rise (mm)	1250
Ring thickness (mm)	330
Arch width (mm)	675
Span : rise ratio	4:1
Shape	Segmental

The second stage is to consider the loading. The backfill was represented by the application of point loads at the quarter points of the span and were equivalent to having 300mm cover to the crown. No horizontal forces were applied to the arch to replicate horizontal backfill pressures. Live loading was replicated by the application of a point load at the quarter point in the case of the static monotonic loading test to failure. In the case of each of the fatigue tests, the live loading was applied as alternating point loads applied to the quarter points to replicate the passage of a vehicle. Fig. 4 shows the test set-up for 5m arches.

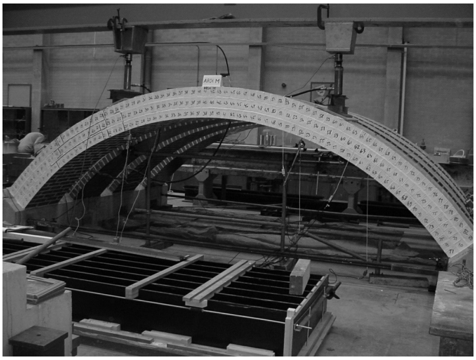

Figure 4 : 5m arch setup

The third stage is to determine the material properties. These are presented in Table 2. The compressive strength of brickwork was determined by five-course brickwork prisms, while the shear strength was determined by triplet tests.

Table 2 : Arch material properties

	Compressive strength (N/mm^2)	Shear strength (N/mm^2)	Density (kg/m^3)
Class 'A' Engineering brick	154	N/A	2370
Brickwork	25	0.3	2180
Mortar: 1:2:9 cement:lime:sand	1.7	N/A	1550

The fourth stage is to undertake a series of analyses that considers each of the possible modes of failure.

In the context of the laboratory tests, holistic considerations ruled out failure by inadequacy of the foundations (and backfill as it was represented by applied loading). Failure by the formation of a mechanism, crushing, ring separation and punching shear all needed to be considered. The brickwork had a static compressive strength of 25 N/mm^2 and so compressive failure was judged to be unlikely as the maximum thrust was calculated to be only of the order of 55kN, i.e. it would require only 3mm of depth of the arch to carry the entire maximum thrust.

A range of fixed, 2-pinned and 3 pinned arch idealisations were analysed using a simple linear elastic FE model (ANSYS 9.0). These gave not only the compressive and tensile stresses but also the longitudinal and radial shear stresses.

The fifth stage is to consider the PLS in the context of the modes of action. If flexure is the criterion (based upon an endurance limit of 50% of the ultimate static compressive strength of 25N/mm^2 and a tensile strength of 0.5N/mm^2) the load limit can be shown to be about 12kN if the arch behaves as a no-hinge arch. (Once the arch develops hinges there is a redistribution of stress that precipitates further hinging and stress enhancement until a mechanism is formed). The endurance load limit for ring separation based upon a longitudinal shear endurance limit of 0.1N/mm^2 (which was determined from a series of laboratory tests) can be shown to be approximately 6kN. There is sufficient radial shear/frictional resistance to prevent punching failure at this level of loading. The above indicates a PLS load limit of 6kN.

The sixth stage is to consider the ULS in the context of the modes of action. A mechanism will form prior to general crushing. By analysis a 4 hinged mechanism gives a collapse load of 35kN. The load to cause longitudinal shear failure (ring separation), using the static shear strength, is approximately 60kN. There is sufficient radial shear/frictional resistance to prevent punching failure at this level of loading. The above indicates a ULS load of 35kN.

From the above simple example, it can be seen that the new approach gives a PLS long term load carrying capacity of 6kN associated with a ring separation mode of failure under fatigue loading. This is significantly smaller than the 35kN load carrying capacity derived for the ULS and is at odds with the current practice of taking half that value i.e. 17.5kN. It is significant to consider this situation in the light of our new knowledge and understanding because although the adoption of 17.5kN as a working load limit would not cause collapse or even early signs of failure, it would significantly reduce the residual life of the bridge from an infinite number of cycles at 6kN to only about 300000 cycles at 17.5kN based upon laboratory test data (Melbourne et al. 2004).

4 DISCUSSION

It is suggested that the SMART method can currently be applied as a methodology which identifies potential critical parameters. The SMART method differs from existing methods in as much as it considers long-term behaviour and attempts to quantify residual life. However, at this stage, the S-N curve type information is only available for specific laboratory-based research programmes and consequently, at present, the application of the method relies on the use of permissible stresses unless specific data are available. It may be that due to the variability in the mechanical properties of the materials and construction details of masonry arch bridges that a probabilistic approach to structural performance might be more achievable.

It should be noted that initially even the most sophisticated FE model will behave elastically at low stress levels. Additionally, once cracking is recorded in the FE model then the assessing engineer should consider the effect that this will have on the residual life of the bridge.

Currently available ultimate limit state analytical techniques can be used to determine the ultimate carrying capacity. However, it is important that the structural idealisation incorporates all the possible modes of failure. If certain modes of failure are not included (e.g. ring-separation, abutment movement, snap-through, etc), then the analysis may over-estimate the carrying capacity.

It is recognised that the example presented in the paper relates to the specific issue of ring-separation in multi-ring brickwork arches; however, it serves to highlight the necessity to consider all modes of behaviour. Movement of abutments, voussoir slippage, internal construction etc all influence the working load stress regime and hence whether or not critical stress levels are being exceeded at normal operational levels of loading. This is very important when considering the residual life of the bridge.

5 CONCLUSIONS

Current assessment methods were considered. Although they were found to be able to predict ultimate carrying capacities with some confidence, serious concern was identified with respect to predicting long term behaviour and residual life.

A new assessment procedure was presented (SMART) which differs from existing methods in as much as it brings together all the existing assessments methods into a single methodology that considers not only the load carrying capacity but also long-term behaviour and residual life. However, at this stage, the S-N curve type information is only available for specific laboratory-based research programmes and consequently, at present, the application of the method relies on the use of permissible stresses. It may be that due to the variability in the mechanical properties of the materials and construction details of masonry arch bridges that a probabilistic approach to structural performance might be more achievable. In any case, either methodology is compatible with the assessment algorithm.

It is suggested that the SMART method can currently be applied as a methodology which identifies potential critical parameters. In the example presented in the paper, the longitudinal shear stress was identified as the critical parameter in the determination of permissible axle loads using a range of simple elastic idealisation and comparing the analytical output with the ultimate load carrying capacity.

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