Sensitivity analysis and parameter identification on historical masonry arch bridges

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ABSTRACT: Close to reality computation of the serviceability and load-carrying capacity of old masonry arch bridges for rising live loads is a current problem. In Germany many of such road- and railway bridges exist. Some of the most important features for applicable numerical evaluation of masonry arch bridges are a realistic structural model (very often three dimensional models are necessary), the consideration of the nonlinear load history and in particular the inclusion of a realistic nonlinear material model applicable for historical masonry.
Often input parameters necessary for a realistic simulation are unknown. Then e.g. material and geometry data must be determined by measurements. In particular at historical masonry bridges these data can show relatively strong variance. By sensitivity studies can be determined correlation between the input data and output value of the computation model. As basis of measuring program so the most important input values can be determined.
Sometimes some input values can be measured only indirectly. In these cases parameter identifications can be used for the determination of the input values and for validating the computation model.
The paper presents powerful strategies for sensitivity analysis with stochastic sampling methods and for parameter identification with optimization algorithms. Several numerical examples from a masonry arch bridge demonstrate the application of these methods.
The masonry behaviour described by the presented practical applications with three-dimensional elasto-plastic continuum models for regular and irregular masonry types. The constitutive models are based on multisurface plasticity theory and includes anisotropic elastic and inelastic behaviour depending on the orientation of the masonry joints. The yield domain, its hardening law and softening law are defined according to experimental results. On this basis it is possible to simulate masonry-specific failure and damage mechanisms.

1 INTRODUCTION

In most cases it is important for the assessment of the stability and the service ability of historical masonry bridges to take material samples from the building and to determine the necessary material codes in the laboratory as basis for verification. For the material examination, there are following fundamental questions:

- Were exactly from the building should the material samples be taken?
- Which material codes are important regarding the verification and therefore have to be secured by experiments?

Sensitivity analysis are just right for answering these questions The localization of relevant areas for taking the material tests can be orientated at the utilization of the structure on one hand and the necessary evidences on the other hand. Thus, it is important to compute the load flow and the different load states within the building. As shown in Schlegel (2004), this can only succeed at masonry with the help of a realistic material model which is capable to describe the essential load relocations and failure mechanisms of the masonry structure. In this article, the FEM calculations are carried out by the FE program ANSYS using especially for regular and irregular masonry types of implemented nonlinear material models under the application of Schlegel (2004).

Furthermore, one has to be aware that the material quality can highly scatter especially at historical masonry. Single material samples can only offer local statements. Sometimes, some parameters can not be determined directly. For the identification of the global system behaviour further measurements are necessary (Zabel & Bucher 2000), for example deformation or modal variables. With the help of modern optimization algorithms, essential global system and material parameters can be identified. The adjustment between measurement and recalculation belongs to the classical tasks of a model validation. If the difference between measurement and recalculation is too big, an optimization task of minimizing the difference can be formulated (Will 2006).Optimization tasks of adjusting measurements and recalculation are often called identification problem (e.g. parameter identification) or inverse problem.

For the sensivity analysis and parameter identification below, the software optiSLang (Dynardo 2006) is used which has been developed by Dynardo.

2 MODELLING MASONRY

Regular masonry types appear in all examples of artificial brickwork and similarly treated natural stone masonry. Because of the regular arrangement of the mortar joints, orthotropic constitutive equations can be assumed. In continuum models, the consideration of different failure-mechanism of masonry (failure of the masonry units, failure of the joints and combined failure can be realised by using a multisurface yield-condition that combines several yield-criteria (Lourenco 1996). Here, for the description of the orthotropic strength of regular masonry types, the yield criterion shown in Fig. 1 is used. This yield criterion based on the work of Ganz (1985).

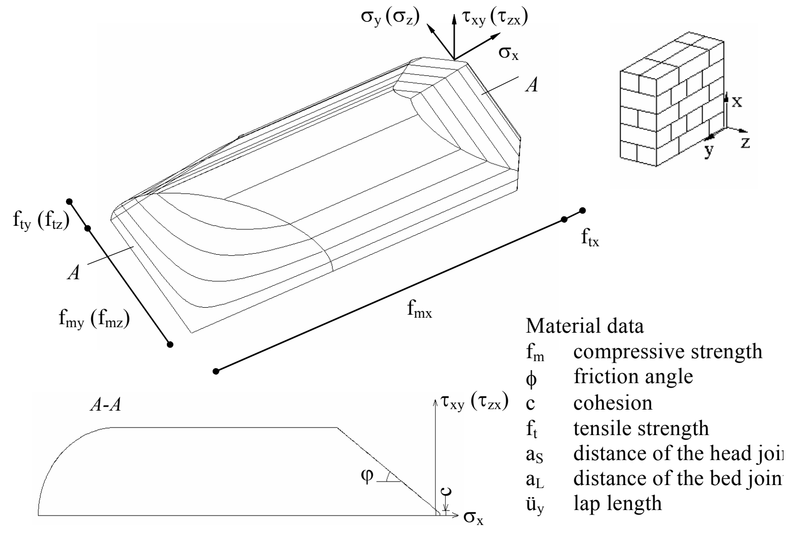

Figure 1: Yield criterion for regular masonry.

Irregular masonry assemblies are characterized by the absence of favoured joint directions. Their crack and deformation-behaviour can therefore better be described by isotropic material-formulations. Here, the Mohr-Coulomb Criterion is used for the description of the strength of irregular masonry. The stress-strain relations used for the description of nonlinear material behaviour are displayed in Fig. 2.

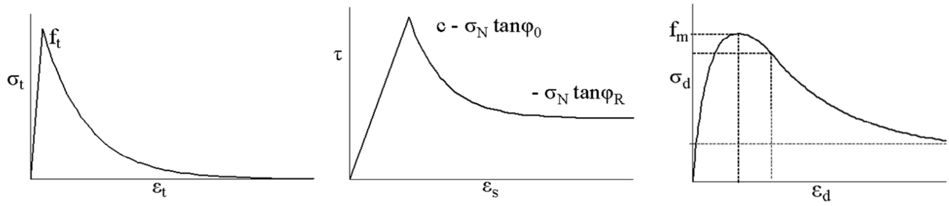

Figure 2: Stress-strain laws for the masonry material models.
(left: tension, middle: shear, right: compression)

For the integration of elasto-plastic calculations, the return-mapping method is used. The plastic multiplicators λ of the active flow criterion i are determined in the equation system Eq. (1) according to the suggestion of Simo (1988):

$$\left\{\frac{\partial F_n(\sigma)}{\partial \sigma}\right\}^T D\, d\varepsilon = \sum_{i=1}^{Set\ activ\ F.} \left[\gamma_n{}^T D\,\frac{\partial Q_i}{\partial \sigma} - \frac{\partial F_n}{\partial \kappa_n}\frac{\partial \kappa_n}{\partial \lambda_i}\right] d\lambda_i \tag{1}$$

with

$$\gamma_n = \frac{\partial F_n}{\partial \sigma} + \frac{\partial F_n}{\partial \kappa_n}\frac{\partial \kappa_n}{\partial \sigma}\text{; activity criterion: } d\lambda_i \geq 0\,.$$

Detailed descriptions of the used material models are included in Schlegel (2004).

3 ANALYSIS OF A MASONRY ARCH BRIDGE

The examined railway viaduct and the different material areas are presented in Fig. 3. The length of the arch bridge is 103,62 m and the height approx. 30 m.

Figure 3: Masonry arch bridge (left), material areas (right)

The load bearing capacity of the structure should be calculated for the actual state and the serviceability of masonry arches should be tested for current life loads. The arches are made of regular sandstone masonry.

The spandrel walls and the piers exist of quartzite schist masonry. The piers normally consist of multiple layers of regular and irregular masonry. No exact materials are defined for the stiffness and the strength of masonry. They should be defined by corresponding tests and material examination. The calculations described below are necessary for the localisation of relevant areas for taking the samples and for the identification of relevant material parameters. Therefore, the order of the material variables is estimated in respect of lower and upper limits with the help of the literature, e.g. Schubert (2000) and different codes (DIN 1053, Eurocode 6, UIC-Kodex 778-3 E).

For the determination of significant life load positions, the train passage over the bridge was simulated by using estimated mean material variables. Fig. 4 shows the most inconvenient life load position for the arch load.

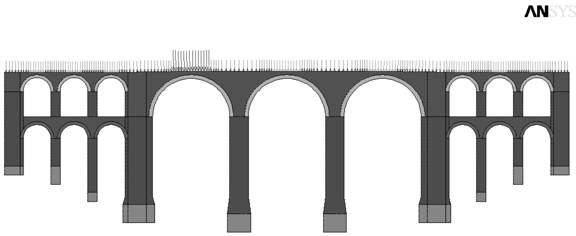

Figure 4: Relevant life load position

The sensitivity analysis is carried out in the service load status and help to determine the failure load. Thus, the service load status is calculated with nonlinear load history calculations with three load steps:

- 1: dead loads
- 2: temperature pressure in winter time
- 3: life load according to Fig. 4

For the determination of the failure load, as follow-up the life load is increased until the bridge failures.

4 SENSITIVITY ANALYSIS

The sensitivity analysis was done for the 21 most important scattered, independent material variables (see Tab. 1) of different material areas which is shown in Fig. 3 (right). These are the e-moduli of the arch masonry ey_1, ey_5, the spandrel walls ex_2 and the pier ex_3, the masonry compression strength fmx_1, fmx_2, the tensile strength ftx_1, ftx_2, ftx_5, the shear strength variables, the friction angle phi and cohesion c, the joint distances of the horizontal joints al_1, al_2, al:5 and the vertical joints as_y1, as_y2, as_y5 as well as the density of the material areas roh_1, roh_2, roh_3, roh_5.

The aim of the sensitivity analysis is the examination of the sensitivity of the response values according Tab. 1 and failure load on the variation of material parameters. It is obligatory for a significant statistic analysis to represent the design space equally, that means an equal representation of the upper and lower bounds of the parameters. Thus, optiSLang offers efficient methods.

In this case, the Latin Hypercube Sampling (LHS) was used as random sample plan (DOE scheme). LHS is a stratified sampling methology which divides the parameter distribution into equal probability intervals. The parameters within the interval are chosen by chance. The LHS technique tries to represent the design space equally by minimising the variation of distance vectors. This secures an optimal random sample density over the entire parameter space for the statistic analysis. Each parameter is supposed to be uniformly distributed. 199 Designs were calculated for the sensitivity analysis.

The statistic sensitivity analysis utilizes linear and quadratic correlation matrixes, its corresponding correlation coefficients and the coefficients of determination of the response parameters. The linear (or quadratic) correlation coefficient can show the degree of an statistic linear (or quadratic) correlation between input and response parameters. It indicates how two databases correspond linearly (or quadratically) and states to what extend they depend on each other linearly (or quadratically).

4.1 Sensitivity of the Serviceability

For the evidence of the serviceability, stresses and deformations are especially interesting. Therefore, the arch stresses and vertical deformations of masonry and the horizontal compression strength of the spandrel wall masonry are chosen as response parameters for the sensitivity analysis (see Tab. 1). The qualitative distribution of vertical deformations are shown in Fig. 5 (left) and in Fig. 5 (right) the arch and horizontal stresses are presented.

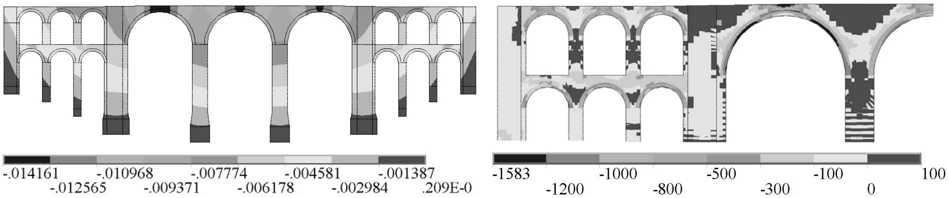

Figure 5: Response values under service load level
left: vertical displacements (m), right: arch and horizontal stresses (kN/m²)

Table 1 : Response parameter

Response value	description
z1	vertical total displacements of masonry arch crown, level 1
z2	vertical relative displacements of masonry arch crown, level 1
z3	maximal arch compression stress, arch level 1
z4	compression stress of masonry arch crown, level 1
z6	maximal horizontal compression stress, spandrel walls

Table 2 : Results for the Response parameter

Response	mean value	minimum	maximum	CoV	CoD lin[%]	CoD quad [%]
z1	-0.0135 m	-0.0177 m	-0.0109 m	0.1	93.0	99.0
z2	-0.0080 m	-0.0094 m	-0.0070 m	0.063	86.0	94.0
z3	-1540.0 kN/m²	-2440.0 kN/m²	-1090.0 kN/m²	0.164	85.0	92.0
z4	-1220.0 kN/m²	-2440.0 kN/m²	-497.0 kN/m²	0.374	96.0	98.0
z6	-1850.0 kN/m²	-2940.0 kN/m²	-717.0 kN/m²	0.294	97.0	99.0

CoV – Coefficient of Variantion, CoD – Coefficient of Determination

The results for the response values are listed in Tab. 2. The linear correlation matrix is shown in Fig. 6 (left). It is a symmetric matrix that includes linear correlation coefficients. The size of the correlation coefficients show relevant material parameters with high influence on the response parameters and can be read from the matrix. In Fig. 6 (right), the correlation coefficients of the material parameters are stated for the vertical relative displacements of masonry arch crown. In Fig. 7 (left) are shown the corresponded coefficients of determination. As expected, E-modul ey_1 of arch masonry has the highest impact on the vertical bending. The linear correlation coeffizient r is 0,73 and the coefficient of determination is $R^2 = 54$ %. That means 54 % of the change of the arch bending can be explained by the distribution of ey_1. Further material parameter with relevant correlation are ex_2 ($R^2 = 10$ %) and phi ($R^2 = 7$ %). In Fig. 7 (right), the linear correlation coefficients of material parameters are stated for the maximal arch compression stress. Here, the three most important material parameters are ex_2 ($R^2 = 49$ %), ey_1 ($R^2 = 38$ %) and phi ($R^2 = 8$ %). As a result, the e-moduli of the arch masonry and the spandrel walls as well as the friction angle are the most important material parameters for the evidence of the serviceability. The resulting correlation between ey_1 and z2 is presented in Fig. 8 (left) as anthill plot. Fig. 8 (right) shows the resulting correlation between ex_2 and z3.

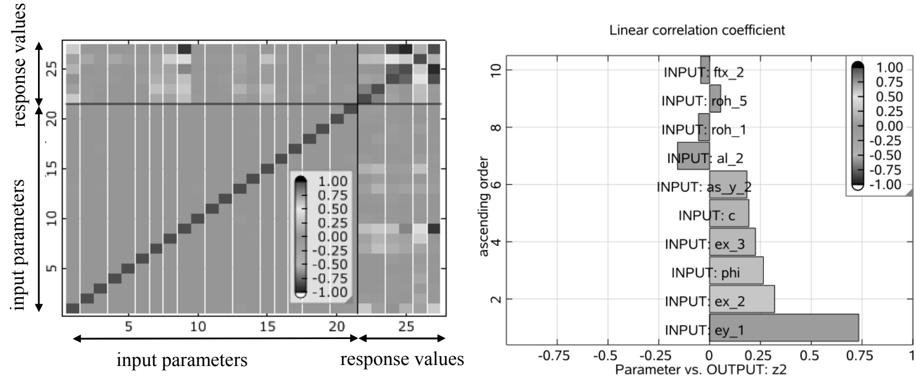

Figure 6: Left: linear correlation matrix, service load level;
right: linear correlation coefficients, response z2– relative vertical displacement of the arch crown

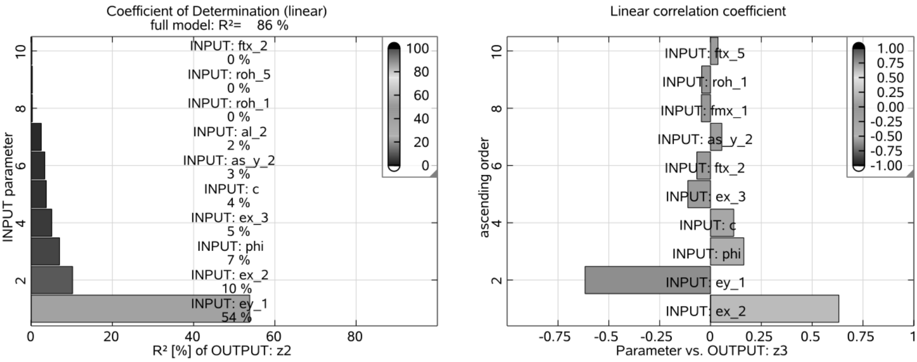

Figure 7: Left: coefficient of determination, response z2
right: linear correlation coefficients, response z3

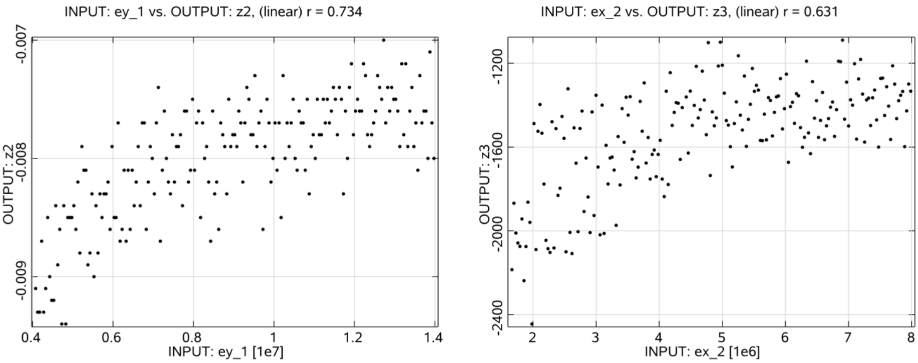

Figure 8: Left: anthill plot response z2 vs. input ey_1 (kN/m²)
right: anthill plot response z3 vs. input ex_2 (kN/m²)

4.2 *Sensitivity of the failure load*

The found failure mechanisms on reaching the failure load are displayed in Fig. 9. It becomes obvious that the failure of the structure is described by a lateral evasion of the arch bearing over the pier, a failure of the spandrel walls and a failure of arch masonry.

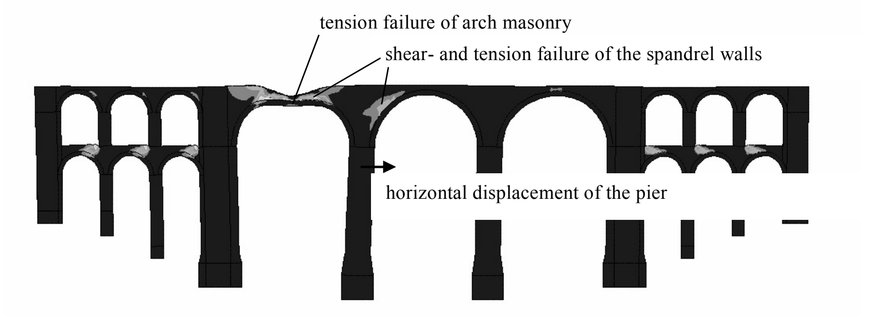

Figure 9: plastic strains and failure mechanisms

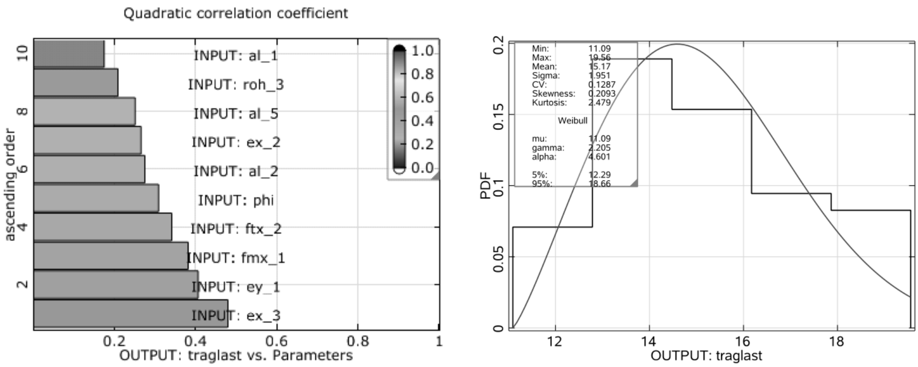

Figure 10: Left: quadratic correlation coefficients of the most important parameters; right: Histogram failure load

The statistic evaluation shows that only 74% of the entire distribution of the failure load can be explained by linear correlation coefficients. For quadratic correlation the coefficient of determination is 96%. Therefore, the quadratic correlation coefficients in Fig. 10 (left) have to be involved. The failure is obviously highly influenced by ex_3 (e-modul of the pier masonry R^2 = 23 %) and with that by the horizontal move of the pier head. Further relevant material parameters are e-modul ey_1 (R^2 = 16 %) and the compression strength fmx_1 (R^2 = 14 %) of masonry as well as ftx_2 and phi for the failure load of arches. Fig. 10 (right) shows the resulting distribution of the response failure load in a histogram.

5 PARAMETER IDENTIFICATION

Based on the results of the sensitivity analysis, the material tests were carried out. Because of the relatively irregular masonry of the spandrel walls ex_2 and phi_2 should be determined by a parameter identification. For this purpose, the results of deformation measurement under a life load of 60 t should be recalculated. The left large arch was loaded. Two measured values were available, the vertical displacement of the crown from the left large arch and the middle arch. An optimization based on evolutionary algorithms was performed with the aim to minimize the difference between measurement and recalculation. The measured displacement under the load amounted to 2,5 mm. This measured value corresponds to the computed relative value of the displacement. Fig. 11 shows the history of optimization solver (left) an the input parameter of the best design in terms of relative size to bounds (right). The modulus of elasticity of the spandrel walls was identified to 1844 MPa and the friction angle to 38°.

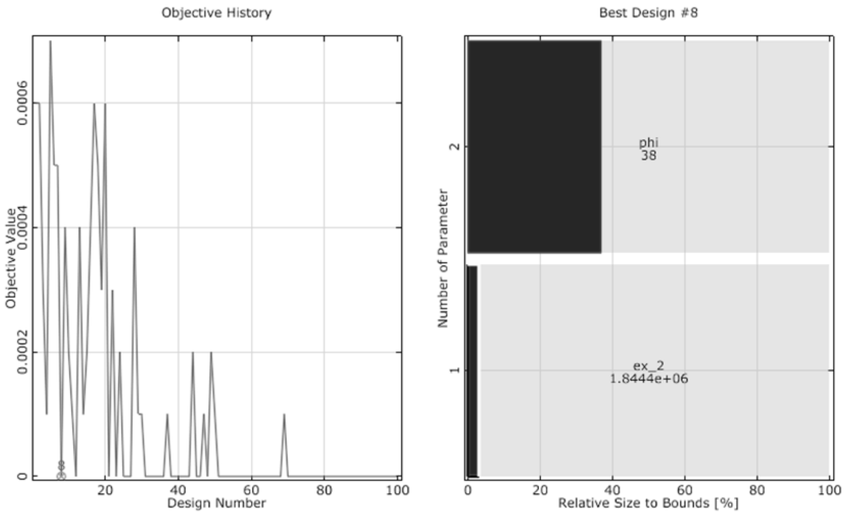

Fig. 11: Left: History of optimization solver; right: values for input parameters of the best design in terms of relative size to bounds

6 CONCLUSIONS

By coupling stochastic sampling methods, modern optimization algorithms and powerful numeric models of calculation, it becomes possible to identify relevant system and material parameter from masonry arch bridges for the assessment of stability and serviceability. The calculation models can be validated through the adjustment of the measurement results and its quality of prediction can be rated.

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