Design and testing of masonry arches: a project of bachelor students in civil engineering

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ABSTRACT: The student project presented here was set up to introduce a group of second year bachelor students to the structural behaviour of masonry arches. The aim of the project was to make them acquainted with the structural behaviour of masonry arches and the influences of external forces and displacement of the abutments on the stability of the arch. Therefore, a dual approach was followed. Firstly, in a theoretical part, the geometry of different types of arches and the influence of the mechanical properties of the mortar and bricks on the bearing capacity of the arch was assessed, using manual calculations and a limit analysis software tool. Secondly, in an experimental part, arches were fabricated and their load-bearing capacity was determined experimentally

1 INTRODUCTION

The arch is an important construction element, certainly in historical buildings and therefore civil engineers might get involved in their structural behaviour during a restoration project. The student project presented here was set up to introduce a group of second year bachelor students to the structural behaviour of masonry arches. The aim of the project was to make them acquainted with the structural behaviour of masonry arches and the influences of external forces and displacement of the abutments on the stability of the arch. Because their knowledge on structural analysis is limited to static structures, the masonry arch is well suited to introduce them in the world of structures and building materials.

This project for bachelor students originates from the research activities performed by the Civil Engineering Department related to the restoration of historical buildings. In that perspective and to clearly indicate the overall framework of the project, an introduction to the safety of masonry arches and vaults was accompanied with a site visit to the Sint-Jacobs church at Leuven. In 2000, the flying buttresses were removed and replaced by tie-rods because of lack of load-bearing capacity (Schueremans et al.,2006; Smars et al., 2006).

The relevant background of the 2nd year bachelor students at their first semester is limited to the compulsory courses of static and a general course on mechanics of rigid bodies. The project itself is worked out during a period of 10 weeks, 5 hours each week, within two teams of 6 students, under guidance of a teaching assistant. Besides technical goals, the objectives of the project also cover project management, written and oral reporting, team work and communication aspects. These are not treated here.

A dual approach was followed. In the theoretical part, covering the initial 5 weeks, the structural behaviour of masonry arches is treated. An answer is given on how external forces are transferred through the arch towards the abutments. The effect of the geometry of different types of arches and the influence of the mechanical properties of the mortar and bricks on the bearing capacity of the arch is assessed. In this part, after an initial seminar on this subject, the students have to design an arch geometry capable of withstanding a central vertical loading. Some geometrical conditions are pre-imposed: a span of 1.40 m and a height varying in between 0.70 and 1.00 m. The focus of the design is on the calculation of the thrustline lying within the cross-section and on the prediction of the overall load-bearing capacity, predicting the central vertical collapse load.

In the experimental part, covering a second block of 5 weeks, 3 arches are fabricated and tested by each group of students, according to their theoretical design. In two of them, a gradual increase of the central vertical loading is applied. In the third one, a gradual increase of the horizontal displacement of the abutments is imposed. The experiments are carried out to get some engineering flavour regarding the actual construction of an arch, the structural behaviour of arches and to investigate the influence of the assumptions made in the theoretical part. During the experiments, the imposed load or displacement and crack openings are recorded. At several locations, the strains are recorded. Standard tests on brick and mortar are performed to back-calculate the material stresses at failure and their relation with the actual load-bearing capacity experienced in practice.

2 STABILITY OF THE ARCHES - THEORETICAL CALCULATIONS

2.1 Thrustlines – manual calculation

There are different levels in assessing the structural safety of masonry arches. As the purpose of the project was to give the students more insight into the structural behaviour of a masonry arch, the stability assessment of the arches of different geometries was based on manual calculations of the thrustlines (Heyman 1980, 1985). For that purpose, the students developed an Excel-worksheet themselves, figure 1a. As the students were able to calculate a thrustline lying within the geometry of the arches they designed, it was assured that the arches would be stable. Additionally, they repeated the calculations, imposing a centric load on top of the arch (F_V) to estimate the theoretical collapse load, Figure 1b for the pointed arch.

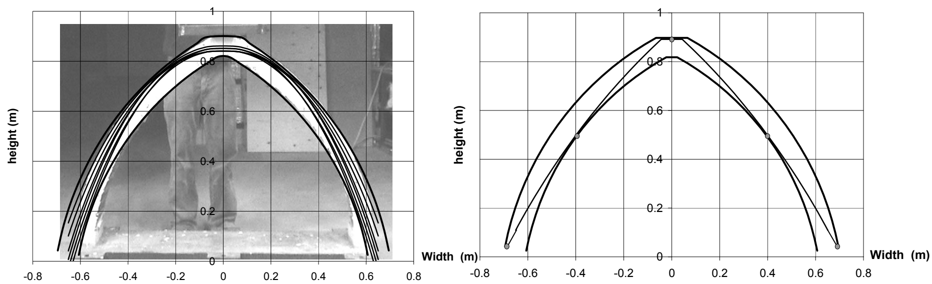

Figure 1: Location of thrustlines within the arch: (a) without external loading; (b) when the maximum vertical loading is applied and the load-bearing capacity is reached (F_V=550 N)

From this manual calculation the students learned that the stability of the arch depends mainly on geometrical issues, that the position of the thrustline is not unique and that several possible lines of thrust lead to different horizontal reaction forces at the abutments.

2.2 Thrustlines - Calipous

The manual calculation providing insight in the structural behaviour was extended using a limit analysis software tool developed in the framework of the Ph. D. thesis of Pierre Smars, called Calipous (Smars, 2000).

The *Calipous* computer program was devised to analyse the stability of masonry arches of complex geometry, possibly subjected to external loads and/or movements of abutments.

From the geometrical point of view, an arch is composed of a set of blocks (b_i) defined as the volume comprised in between two bed joints (j_i and j_{i+1}). A Joint (j_i) is assumed to have a rectangular shape and is defined by a point on the intrados (u_i, v_i), a thickness (t_i), a depth (d_i) and an orientation (φ_i). The set of points (u_i, v_i) on the intrados have all to lie in a vertical plane but the position and orientation of this plane can be arbitrarily set in 3D. If they do not receive a specific value, t_i and d_i remain constant and the orientation φ_i is set perpendicular to the intrados.

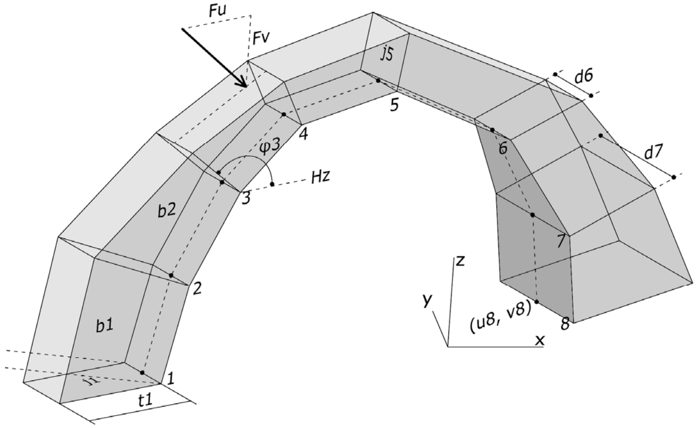

Figure 2: schematic view of an arch as defined in the *Calipous* computer program

External loads can be applied on the extrados of the arch (and possibly on its intrados). Forces can be oblique but they have to act in the plane of the arch (F_u, F_v). The dead weight of the arch is automatically calculated and is always applied. The specific mass of the blocks is assumed to be constant. Arches made of blocks of different materials can be modeled giving pseudo-depths to the corresponding blocks.

The relative eccentricity e_i of a thrustline in a joint is the position of the intersection of the resultant of the forces acting on the bed joint of a block and of the plane of the joint relative to the centre of the section: its value is 100% if the thrustline passes by a point on the extrados and -100% if it passes by a point on the intrados. For an admissible thrustline, the eccentricity has to be comprised between -100% and 100% in all the joints, i.e. the line has to stay completely inside the shape of the arch (Heyman 1980).

The basic calculation is the determination of a thrustline passing by three given points. More interestingly, optimisation routines allow the calculation of 'extreme' thrustlines (maximum and minimum) and 'average' thrustline (minimizing the sum of the square of the eccentricities).

When a thrustline is calculated, graphical results are sent to a graphic window and text results to a text window, from which they can be exported to DXF and text files. The numerical results are the forces F_u and F_v at the abutments and, for each of the joints, the value of the normal and tangential forces N_i & T_i, the relative eccentricity e_i, the angle of incidence θ_i and an estimate of the maximum stress σ_i in the joint (assuming that the joints do not resist to tension, that the stress distribution is linear and that the section is rectangular).

Calipous can also be used to study the stability of an arch subjected to movements of its abutments but this facility was not used in the framework of this project. A special interface was developed (by dr Ann Hendricx) to work directly from within AutoCAD and to have the graphical results sent to the AutoCAD drawing. A batch version of the program was also used for a probabilistic analysis of the stability of masonry arches (Schueremans et al. 2001).

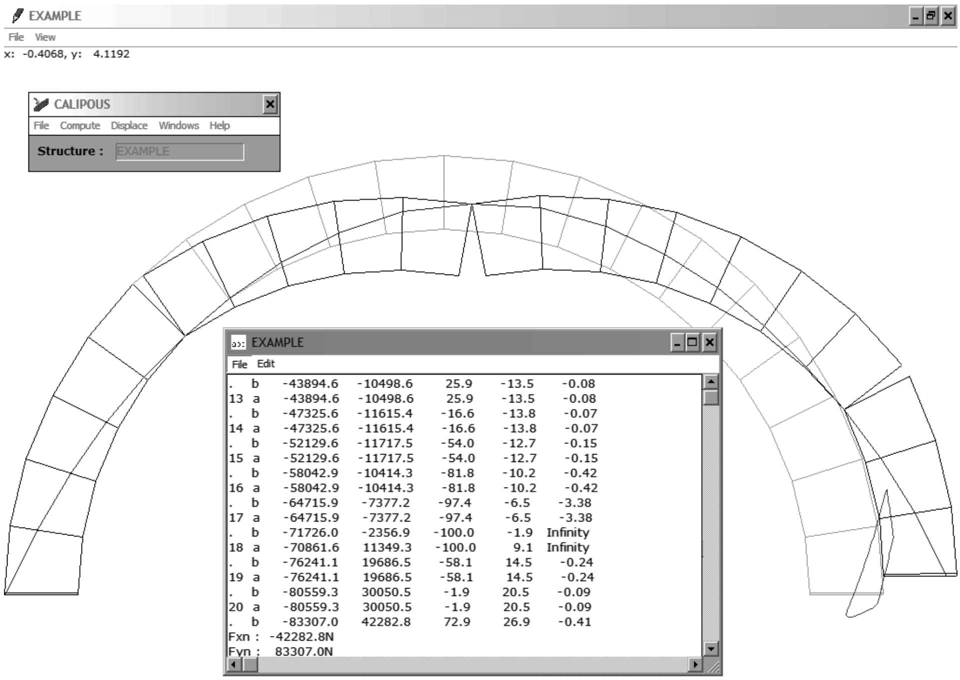

Figure 3: Analysis of an arch with the *Calipous* computer program.

The program was originally developed in Delphi programming language (Borland). A new C++ version is under development.

Based on their analysis of the structural behaviour of masonry arches, the two groups of students chose a pointed arch and a parabolic arch respectively. As arguments to choose a pointed arch, the first group indicated that this type of arch would be better capable to bear the centric load, imposed in the top of the arch during the two loading tests. To divide the centric load, they suggested using a keystone. The second group preferred a parabolic arch, as this type of arch is easily describable in an analytic way, approaches the form of a catenary's curve and has a smaller horizontal component at the abutments.

Using both the manual calculations and Calipous, the two groups calculated a maximum centric load of 550 and 570 N for the pointed and parabolic arch respectively. Hereby, the course of the extreme thrustlines were analyzed, giving special attention to the points where the thrustline touches the intrados and extrados of the arch, as these are the places where the first cracks in the brick-mortar interface will occur.

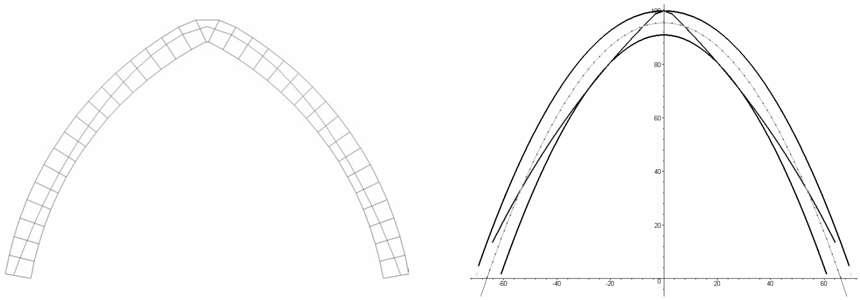
Figure 4: mean thrustline in pointed arch and thrustline at collapse load on the parabolic arch.

3 EXPERIMENTAL RESEARCH PROGRAM

3.1 Description of the program

During the experimental part of the project, the arches were built and tested by the students. First, a wooden centering was constructed for both types of arches in such a way that it could be re-used multiple times. In the following weeks, three arches were constructed as well for the pointed as for the parabolic arch, figure 5. Each of the arches was subjected to a different test:

Figure 5: Construction of arch – limewashing of arch

- A loading test, during which a gradually increasing, centric load was applied until failure;
- A second loading test, comparable with the previous one, the difference being that the arches were constructed with thin plastic foils in between the mortar and the bricks;
- A displacement test, during which a horizontal displacement at the abutments is imposed.

For arches in buildings, a load test with a centric load is not immediately related to practical cases, as external forces are usually low and do not change. However, this load condition is chosen here, as it is a simple static condition, best suited for the purpose of the project, namely the verification of the numerically obtained load-bearing capacity.

For the second loading test, thin plastic foil was placed in between the mortar and the bricks, in order to immobilize the tensile strength of the mortar. This was done to reproduce the conditions assumed during the calculations of the thrustlines, being: tension can not be mobilized within the mortar and bricks of the arch; the compressive strength of the material equals infinity and there is an unlimited friction in between the mortar and bricks. So the arch will only fail if at least 4 hinges are formed and the structure becomes a mechanism.

To impose the horizontal displacement on the arch's abutments, the base of the arch was built on a movable steel profile, allowing for gradually in- and outward displacement, figure 6.

Figure 6: Pointed and parabolic arch, and setup for movable abutment

Before the tests were performed, the arches were limewashed in order to facilitate the observation of occurring cracks and fixed points were placed on one side of the arch in order to follow the increasing width of the cracks when the load or displacement increases by measurements with a mechanical strain gauge.

3.2 Results of loading tests

During the loading tests, initially, 3 hinges were formed as was predicted by the model: one hinge was formed in the top of the arches at the extrados and two other ones at the intrados, Figure 7. For both types of arches, the prediction of the location of the extreme thrustline was rather accurate. As the keystone of the pointed arch was a stronger element, the cracks did not occur in the top, but rather asymmetrically. For the parabolic arch, the location of the hinges was predicted well by the model, see also figure 4, which enabled the use of the strain gauge to measure the development of the crack width.

Figure 7: crack opening in the top of the arch – formation of 4 hinges at collapse of parabolic arch.

The results of the loading tests are summarized in Table 1. By comparing the loading tests of the arches with and without thin plastic foils between the mortar and brick, the influence of the tensile strength of the mortar became visible: The parabolic arch with immobilized tensile strength could bear a centric load of 620 N, which is very near to the calculated maximum load, while the same arch without thin plastic foil could carry a higher maximum load of 871 N before collapse. For the pointed arch, a premature failure occurred in one of the tests due to mishandling of the loading during the test.

Table 1: Experimental results of central vertical loading tests

Arch type	Theoretical load-bearing capacity [N]	Experimental load-bearing capacity [N]	
		No plastic foils	Plastic foils between brick-mortar
Pointed arch	550	638	/ [premature failure]
Parabolic arch	570	871	620

From the results however it is clear that the actual load-bearing capacity very much is in line with the theoretical prediction, certainly when the effect of the adhesion of the mortar is immobilized by means of the plastic foils. The effect of tensile strength of the mortar in the arch leads to a slight increase of the load-bearing capacity. The additional load-bearing capacity is in line with the tensile strength of the mortar, obtained on standard mortar beams, according to EN 1015-11, table 2. The age, composition and curing conditions of the samples correspond with the age at which the arches have been tested.

Table 2: Mortar – test results of tensile strength and compressive strength on 6 standard mortar beams – results given: average, standard deviation ( ); cov [ ], *according to EN 998-2

Mortar composition	Binder [kg/m³] Portland cement	Tensile strength f_t [N/mm²]	Compressive strength $f_{c,m}$ [N/mm²]
Pointed arch – M5*	300	2.07 (0.57) [27%]	6.12 (1.66) [27%]
Parabolic arch – M15*	400	4.16 (0.78) [19%]	17.47 (3.78) [22%]

From these results, for example for the pointed arch, the maximum eccentricity of the thrustline can be obtained using a NTM brittle material model, Figure 8. In this case, an eccentricity of 99% of half the thickness of the bricks is required to obtain the compressive stresses equal to the compressive strength of the mortar. The latter is confirmed by the large eccentricities observed in the hinges during the experiments; see also Figure 7 and Figure 9. In addition, from this analysis, it is clear that the assumption of an infinite compressive strength to calculate the thrustlines is acceptable.

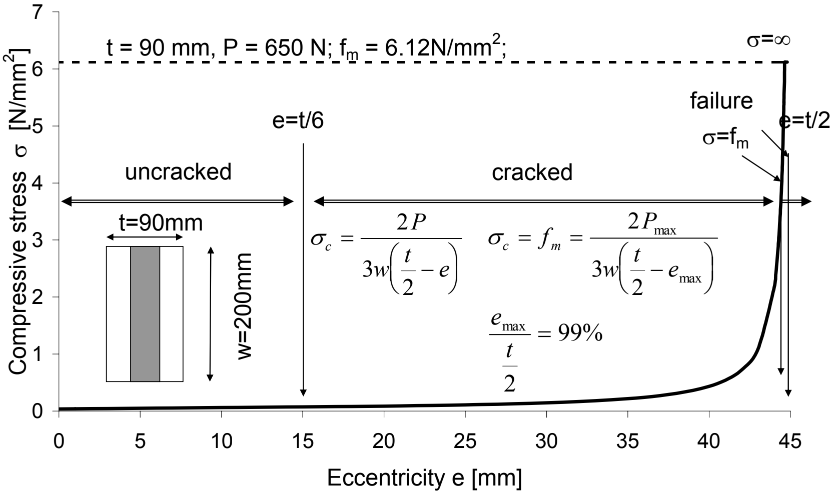

Figure 8: Compressive stresses in the mortar as a function of the eccentricity e of the thrustline in a cross-section for the material properties obtained for the pointed arch, using a Non-Tension Material behaviour

3.3 Results of displacement tests

During the displacement test, one of the arch's abutments was horizontally displaced over a distance of 20 cm, which appeared not to be enough to enforce the arches to collapse. Without external loading, the arches were still stable even though they showed very large deformations and crack openings. This experiment demonstrated the actual capacity for the absorption of differential settlements of the abutments in reality.

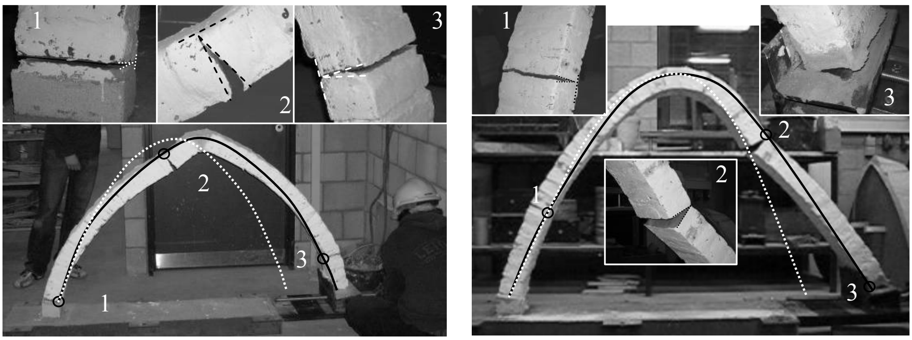

Figure 9: pointed and parabolic arch during displacement test, showing an asymmetric crack pattern.

3.4 Scale effects

The arches built for the tests are relatively small: they have a span of 1.4m. For larger arches, the influence of the tensile strength of the mortar will be lower. On the other hand, for very large arches or for arches with important external loads, the assumption of a maximum eccentricity of 100% can become significantly unsafe.

4 CONCLUSION

This paper focused on the project, set up by The Department of Civil Engineering, Building Materials and Building Technology Division, and performed by an enthusiastic group of students. It presents a possibility to transfer research knowledge on the structural behaviour of arches and masonry towards bachelor students in civil engineering and to stimulate their enthusiasm in this field of research.

For the following academic year, some improvements will cover the shortcomings experienced for this year:

- The displacement test will take place with half of the maximum vertical loading, to have a combined action of the loaded arch and differential settlements;
- The load application point is not in a critical position for the moment, but originates from reasons of symmetry, allowing half an arch to be modeled;
- Different geometries and material properties can be proposed;
- A scale model of an arch bridge with uniform distributed vertical loading is considered.

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