DYNAMIC ANALYSIS OF A FOOTBRIDGE OVER THE S8 EXPRESSWAY ON THE BASIS OF THE RESULTS OF IN SITU TESTS

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SUMMARY

Non-destructive diagnostics for structure may use dynamic measurements realized during vibrations of a facility induced by dynamic environmental impact, enforced by the use of inductors or impulse impact. Dynamic tests and the analysis were undertaken for a footbridge situated over the expressway S8. The studies in situ allowed for dynamic characteristics to be determined: frequency, shape modes and damping for respective frequency of vibrations. High compliance of analytical and experimental results confirmed high quality of real representation of a system with MES model and the undertaken studies. The presented procedure of the research is one of many possibilities given by advanced diagnostics of engineering structures.

Keywords: *Footbridge, in situ tests, dynamics analysis, modal shapes.*

1. INTRODUCTION

According to common definition, a footbridge is a bridge which is intended for the use of pedestrians and cyclists. The term is often meant as small and simple structure, but often they are facilities of significant span and complex static schemes [1].

Urban bridges for pedestrians constitute the challenge for architects, constructors and builders, as the designs of such facilities, most often - landmarks, require the coherent combination of functional and aesthetic functions.

Until quite recently footbridges were of simple static schemes and solutions in terms of materials and structure. They were calculated in a simplified manner on static loads in a border state: vertical – by crowd of pedestrians and horizontal - by wind. However, dynamic measurements were based on the calculation of frequency and mode shapes of the structure and probability of resonance to occur.

Nowadays, footbridges present more and more advanced solutions in terms of structure and materials, thus they become more slender and lighter and more prone to vibrations caused by wind and crowd. Modern footbridges made of light materials are more durable, or those pre-tensioned have smaller cross-sections or they are of comparable cross-sections but bigger spans. Due to that fact, the decrease of spatial stiffness is more significant than the decrease of mass, which results in lower frequency of mode shapes and easier capability for the structure to enter vibrations.

All these facts allow for conclusion that apart from static analyses of the system, it is necessary, just at a design stage, to perform detailed dynamic and aerodynamic analyses for these facilities. [2-5].

2. ANALYSED FACILITY – FOOTBRIDGE MODE SHAPES OVER AN EXPRESSWAY S8

2.1. Facility description

The subject of the dynamics analysis is a footbridge situated over an expressway S8 in km 194+353,04 (Fig. 1). The facility was made within the construction of the expressway S-8 on the section: Walichnowy interchange – Wrocław interchange (A1). The bearing structure is a single-span arch system of 50.0 m span, combined of two parallel arch beams and suspended with vertical hangers, a reinforced concrete platform, performed according to design data with concrete class C35/45. The cross-section of an arch beam is made of circular pipe of 660.4 mm in diameter and same thickness on arch length equalling 20 mm. In addition, the arches are joined by five pipe bracings of 323.9 in diameter and thickness of 10 mm. The cross-section of a platform consists of two beams of 0,60 m in height, joined with a board of thickness min. 0.20 m. Total width of the platform is 4.5 m, but its usable part is 3.0 m.

2.2. Description of a model

Numeric model of a footbridge is made in SOFiSTiK program [6] (Fig. 1). The model required three finished elements: BEAM, QUAD and TRUSS. Numeric model consists of a grid of 2772 nods with: 2556 coating finished elements type QUAD, 676 finished beam elements type BEAM, 14 finished elements type TRUSS.

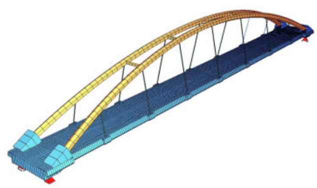

Fig. 1. *Arch footbridge mode shapes over the expressway S8 [photo MSC. Eng. Maciej Malinowski] altogether with visualisation of numeric model [6].*

2.3. Description of in situ tests

The analysed footbridge underwent trial load receiving tests. The research done in September 2013 by a research team from Laboratory of Field Studies in Gdansk University of Technology [8] included two parts: statistical research and dynamic tests.

Dynamic tests included measurements of vertical displacements of a bay, increase of strain (stress) in hangers and acceleration of a platform and arch carrier. Dynamic measurements were taken during the tests, i.e. free march of a group of 6 and 12 people, synchronized march of 6 people and free run and synchronized run of these groups.

In order to perform further modal analysis of the footbridge, the results of vertical acceleration were taken into account a_z [m/s^2] for the structure of the platform. Dynamic measurements of acceleration of the platform were made in all axes of connection of hangers on one side of the structure because of the symmetry of the footbridge in longitudinal direction. The location of measuring points was presented on Fig. 2.

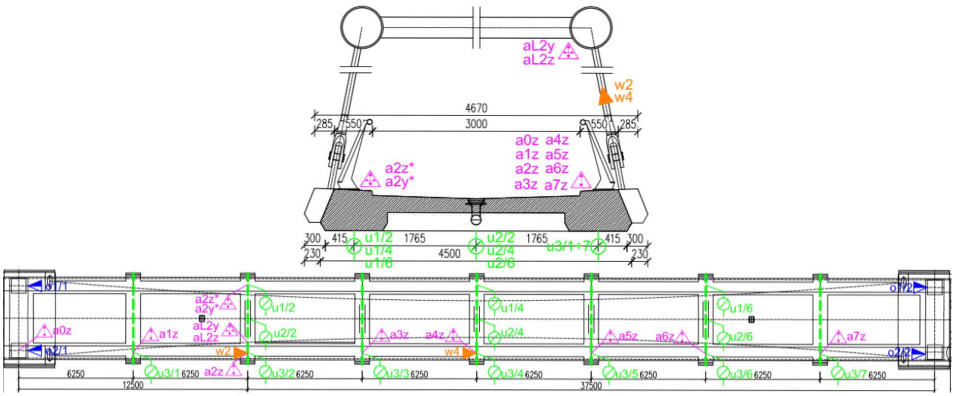

Fig. 2. Location of measuring points [7].

For purpose of the research, the measurements of acceleration course were also taken for dynamic impact test of a container filled with water, dimensions 1x1x1 m, from the height of ca. 6 cm in a central axis of a platform in the axis of hanger no. 1, 2 and 3.

3. THEORETICAL BASIS

3.1. Experimental modal analysis

One of the main goals of scientific research, design and construction of engineering facilities is to provide proper load bearing capacity, safety and proper criteria of use [9]. These are possible due to, among others, the use of appropriate method of modal analysis which allows for determination of dynamic properties: natural frequencies and respective mode shapes and ratios of damping.

The experimental modal analysis is based on controlled enforcement of vibrations at a known force level, after which the measurement of the reception by the facility is taken. It is most common to use impulse test with the use of modal hammer to generate short-time impulse allowing for vibration and then registering the received acceleration with accelerometers.

3.2. Function of frequency response for systems of N-freedom

The equation of motion for a system of N–degrees of freedom may be written according to the matrix equation below (1):

$$M\ddot{u} + C\dot{u} + Ku = p(t), u(0) = u_0, \dot{u}(0) = \dot{u_0}, \tag{1}$$

where M, C, K mean respectively matrix of mass, damping and stiffness of NxN dimensions.

Performing Laplace's transform for an equation (1) results in an equation (2), where a complex variable "s" occurs and at zero starting conditions, it looks like below:

$$(Ms^2 + Cs + K)U(s) = P(s), \tag{2}$$

The equation (2) in the field of frequency, assuming $s = i\omega$ gives the equation as follows (3):

$$H(\omega) = [M(i\omega)^2 + C(i\omega) + K]^{-1}, \tag{3}$$

In which H(ω) means matrix of spectral transmittance.

In order to receive one line of the matrix H(ω), the measurement is taken for the signal of force in particular points of structure, however the response e.g. acceleration is measured in one point. For the matrix column H(ω), signal of force is measured in one point and the value of acceleration is read from all measuring points. The determination of mode shapes is possible at any line or matrix column H(ω).

Additionally, the module of spectral transmittance function |H(ω)| gives information about particular resonance frequency ω_n and determines values of damping with the method of half power (HPM) (4):

$$|H(\omega_1)| = |H(\omega_2)| = \frac{|H(\omega_n)|}{\sqrt{2}}, \xi = \frac{\omega_2 - \omega_1}{2\omega_n} \tag{4}$$

3.3. MAC criterion

MAC criterion [11], [12] provides for comparison between mode shape which were experimentally determined ϕ_{exp} and mode shape received from numeric calculations ϕ_{an} (5):

$$MAC = \frac{\left|\Phi_{exp}^T \Phi_{an}\right|^2}{(\Phi_{exp}^T \Phi_{exp})(\Phi_{an}^T \Phi_{an})}, \tag{5}$$

The values received from the equation above are between zero and one, but the lack of adjustment of shapes is proved by values close to zero, however, ideal adjustment means values close to one.

4. MODAL ANALYSIS OF FOOTBRIDGE

4.1. Damping value (ξ) for footbridge

In order to perform modal analysis for footbridge the level of damping ξ for the structure must be determined through numeric calculations. On the basis of the registered during trial load acceleration for the platform of vertical force a_z, in the axis of respective hangers induced by free vibrations caused by synchronized jumping of 6 people, the value of average logarithmic damping decrement was received.

Knowing the real level of damping for the facility, which equals $\xi \cong 1.4$ %, the coefficients of proportional damping matrix a_0 and a_1 were determined, which are necessary for damping numeric calculations in DYNA module of SOFiSTiK program. For that purpose, numeric frequencies and mode shape were determined. Numeric frequencies and mode shape were determined for the footbridge (Fig. 3).

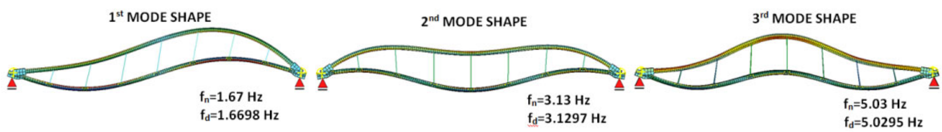

Fig. 3. *The set of received numeric frequencies and mode shapes for the footbridge [6].*

On the basis of matrix equation [10] the coefficients of proportional damping matrix a_0 and a_1 for f_1=1.67 Hz, f_2=3.13 Hz, $\xi_1 = \xi_2$=1.4% equalling a_0=0.175 and a_1=0.00101 were determined.

4.2. The analysis of in situ tests results

4.2.1. Description of the research

The footbridge was analyzed dynamically through impulse impact caused by the container filled with water in the axis of structure symmetry. The analysis in axis of hanger 1 was selected as a representative case. The analysis with the use of containers did not incorporate pre-programmed signal of dynamic load, thus the dynamic impact of a container into a platform had to be modelled.

4.2.2. Verification and validation of calculation model

In order to perform validation of the model the comparison of frequencies between mode shapes was made and they were numerically determined with SOFiSTiK program and separated on the basis of given FFT registered during trial tests, i.e. during the impact of the container into the platform, course of vertical acceleration for the bay (Tab. 1).

There was the need to find the reason for differences in the received frequencies of a shape mode. The verification of materials and cross-sections used for building the model was made in terms of initial assumptions for the model – thus, there was a change, against initial assumptions, of a concrete class for C70/85. The concrete parameters were estimated on the basis of laboratory tests results for a material inbuilt into a facility.

Tab. 1 presents frequencies of shape mode received before (1*) and after (2*) validation of numeric model.

Table 1. *The set of received frequencies for a natural shape mode.*

Mode Number	$f_n(1^*)$ / $f_n(2^*)$ [Hz]	$f_d(1^*)$ / $f_d(2^*)$ [Hz]	$f_{d,measured}$ [Hz]
1	1.52 / 1.67	1.5199 / 1.6698	1.75
2	2.89 / 3.13	2.8897 / 3.1297	3.24
3	4.75 / 5.03	4.7495 / 5.0295	4.93

The presented results show that the numeric model is quite-well adjusted to real movements of footbridge structure over S8. The relative error for subsequent frequencies of mode shapes equals f_1 = 3,70%, f_2 = 3,58% and f_3 = 2,18%.

4.2.3. The analysis of in situ tests

For the analysis of trial in situ tests for loads of the facility the impact from a container was replaced by a triangular impulse. At this stage, the weight of the container influencing the structure was not taken into account as it is insignificant for dynamic characteristics of the structure (Tab. 1).

To introduce triangular impulse, the maximal value of force was determined for the container to hit the structure (F_1) and the time of contact between the container and the structure, as well. (t) [6].

Fig. 4 presents impulse enforcement put on the axis of structure symmetry in axis of hanger 1 and the exemplary course of acceleration for the nod situated in the right axis where the hanger no. 5 is connected.

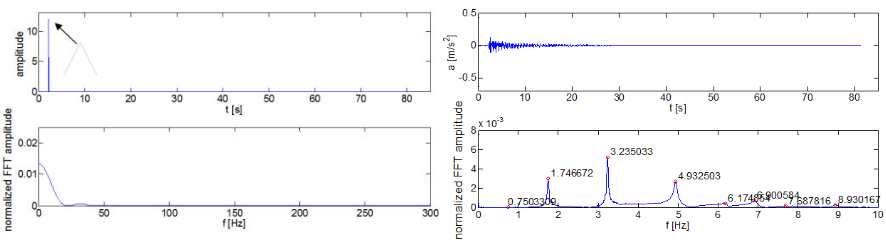

Fig. 4. *Time line of the impulse and acceleration altogether with FFT [6].*

The graph FFT above shows striations reflecting three analysed frequencies of mode shapes of a beam: f_{1d} = 1.747 Hz, f_{2d} = 3.235 Hz and f_{3d} = 4.933 Hz.

The next step included the determination of particular elements of the matrix of spectral transmittance function. The impulse was put on the axis of structure symmetry in the axis of a hanger no. 1, however the results of acceleration were registered in nods situated in the right axis of connection of particular hangers. Measured signals of acceleration were transformed to the field of frequency with the use of FFT. Fig. 5 presents the course of FFT for all measuring points.

Then, the matrix of spectral transmittance H(ω), presented also on Fig. 5, was received through division of acceleration signal by force signal.

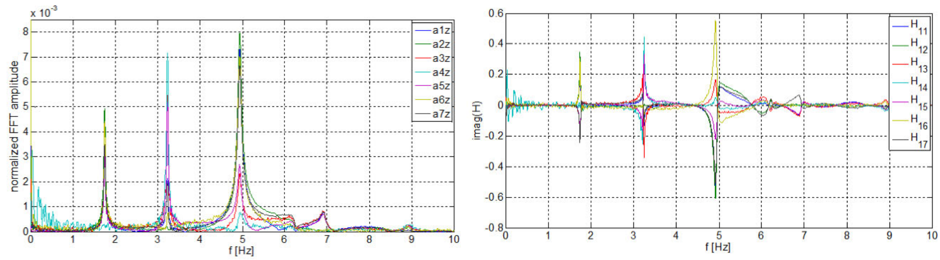

Fig. 5. *FFT corresponding to particular measuring points and matrix H(ω) [6].*

On the basis of the spectral transmittance function H(ω) the course of shape mode was determined.

4.2.4. Numeric analysis

Numeric analysis was undertaken similarly to the modal analysis for the in situ tests. However, in numeric analysis the impact caused by a container was simulated by concentrated mass. In module SOFIMSHA of numeric program the weight of the container was implemented as concentrated mass equalling 1 t (Fig. 6). It was also taken into account in module ASE to determine the frequency of structure shape mode.

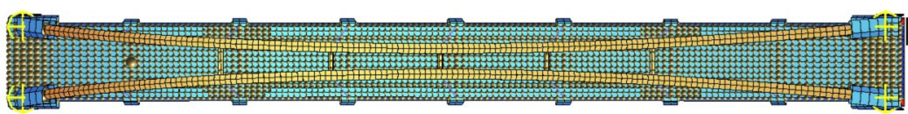

Fig. 6. *Visualisation of numeric model altogether with the implemented concentrated mass. [7].*

Tab. 2 presents frequencies of mode shapes received before (1*) and after (2*) modification of numeric model.

Table 2. *The set of received frequencies of mode shapes of the structure.*

Mode Number	**f_n(1*) / f_n (2*) [Hz]**	**f_d(1*) / f_d(2*) [Hz]**
1	1.67 / 1.67	1.6698 / 1.6698
2	3.13 / 3.13	3.1297 / 3.1297
3	5.03 / 5.01	5.0295 / 5.0095

The graph of the applied impulse was presented on Fig. 4, however, exemplary course of acceleration was presented on Fig. 7 for the node situated in the structure symmetry in the axis of the hanger no. 5.

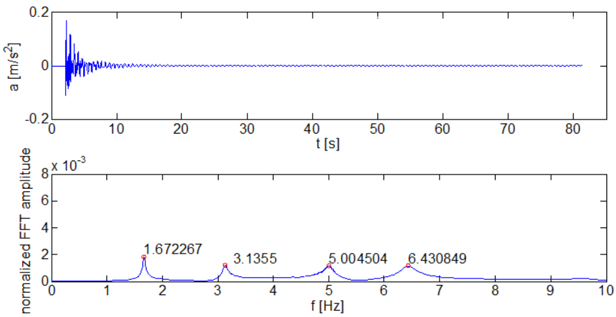

Fig. 7. Time line for acceleration altogether with FFT [6].

Then, again, the particular elements were determined for the matrix of spectral transmittance and the function itself H(ω) (Fig. 8).

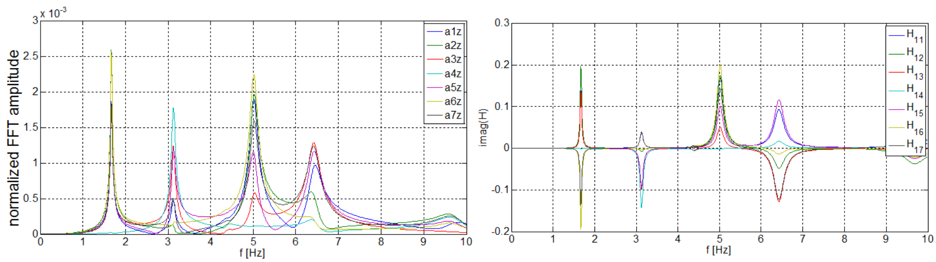

Fig. 8 FFT corresponding to particular measuring points and matrix H(ω) [6].

4.3. MAC criterion

In order to verify the compliance of mode shapes received on the basis of the determined matrix H(ω) in MATLAB program for results of numeric calculations and in situ tests, the criterion of MAC was checked. Fig. 9 presents the course of compared mode shapes and the received matrix MAC, altogether with its spatial graph.

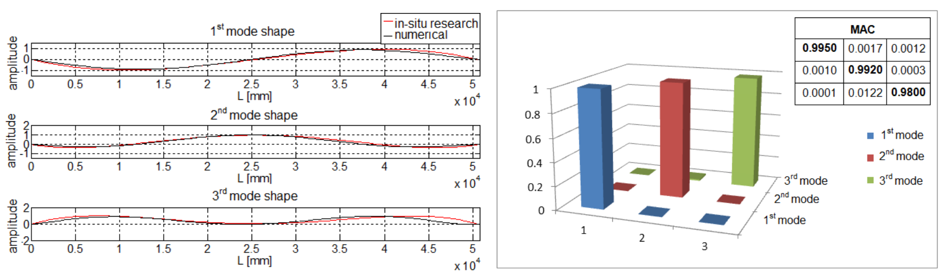

Fig. 9. The course of compared mode shapes for the facility and matrix MAC [6].

The highest accuracy 99.5% was received by the determination of the first shape mode. The accuracy of the second and third shape mode are also at the high level, close to 99%. Compliance of numeric calculations and the results of in situ tests is very high. Taking into account the received results, the conclusion occurs that the modal analysis for footbridge situated over S8 brought satisfactory accuracy of results.

4.4. The determination of modal number of damping on the basis of in situ tests

In order to verify real level of modal damping the imaginary part was determined for the function of spectral transmittance and applied the method of half power and the graph of the module of spectral transmittance function. (Fig. 10). The number of damping was verified for the first and second mode shape.

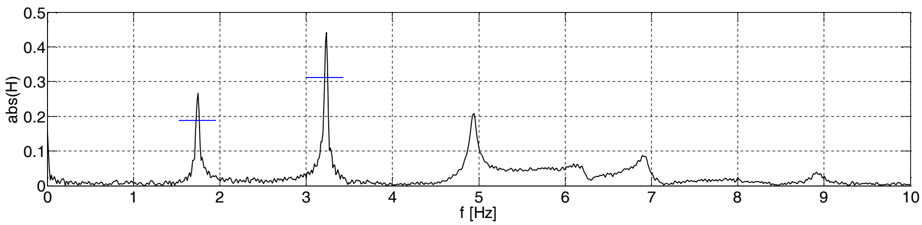

Fig. 10. *Model of the function of spectral transmittance [6].*

On the basis of the equation (4) the first and second number of damping was determined which respectively equal ξ_1=1.06% and ξ_2=0.57%.

While comparing the value of modal damping, received with the use of LDT, and the value on the basis of HPM, quite significant difference between these results was found. The values estimated with HPM method of modal parameters may be flawed. The method is often applied as auxiliary, since for modes of similar frequencies it gives results significantly flawed. While using LDT it is problematic to indicate the time section with freely disappearing shape modes and to determine the number of amplitudes selected for the estimation. For the said facility the number of amplitudes might have been too small, and that is why the error of the method may stem from.

There are a lot of more accurate techniques for the identification of parameters for damping e.g. SSIT (technique for identification of stochastic subspace) or ERA (solution of eigenproblem).

5. CONCLUSIONS

Within the presented modal analysis in MATLAB language, the program for the determination of dynamic characteristics for bridge structures was created. On its basis it is possible to identify, among others, frequencies, shape modes and damping ratios referring to particular frequencies of shape modes.

High accuracy of analytical and experimental results confirmed high quality of analytical representation of the system with MES model and the performed research. The presented procedure of tests is one of the possibilities of advanced diagnostics of engineering structures.

The work hereby confirms significance of simultaneous performance of research in situ supported by theoretical apparatus of well-identified parameters of calculation model. The results of such research and calculations are the basis for proper assessment and the analysis of behaviour of each structure at the time of its exploitation.

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