Castigliano based analysis of single span masonry arch bridges using a spreadsheet

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ABSTRACT: One of the principle problems with the acceptance of new approaches to bridge assessment is the combination of suitable pre and post processors, to allow easy application and interpretation of the results and an acceptable analysis 'engine'. The move from assessments based solely on ultimate load towards assessments undertaken on the basis of serviceability requires the development of suitable 'transparent' elastic based assessment tools.

This paper contains details of the application of Castigliano energy thinning method to the analysis of single span masonry arch bridges using a standard spreadsheet. The use of a standard spreadsheet allows the assessment engineer to have full access to all the elements of the analysis within the assessment process and this transparent approach can be used to simulate either a load controlled ultimate load analysis or can be used to simulate the passage of a load train over the structure and thus find the critical load location. The paper details the main elements of the analysis and presents an example application with the standard graphic output from the spreadsheet.

1 INTRODUCTION

One of the principle problems with the acceptance of new approaches to bridge assessment is the combination of suitable pre and post processors, to allow easy application and interpretation of the results, with an analysis 'engine' which is well understood and trusted by the assessment engineer. FE analysis is a well accepted 'black box' analysis but may be considered as 'overkill' for routine assessments. Limit Analysis based assessment methods can be 'transparent" but are generally quite simplistic. The move from assessments based solely on ultimate load towards assessments undertaken on the basis of serviceability requires the development of suitable 'transparent' elastic based assessment tools.

The use of purpose written 'black box' software has been seen as a drawback. The spreadsheet considered in this paper attempts to rectify this issue by providing an approach that includes all the features of an advanced cracking elastic analysis, using a Castigliano approach, within a standard spreadsheet format, familiar to most practicing engineers. The intention being to allow engineers to understand bridge assessments using a procedure that is both fully transparent and, importantly, suitable for subsequent user modification.

Spreadsheets are in common use by engineers for both tabular based calculations and also for specific sequential type calculations. The application of this Castigliano method spreadsheet contains significant portions of both types.

The spreadsheet as presented can operate in two ways. Firstly, with the user entering the position of the load, or secondly, using the built-in optimisation software, contained within the spreadsheet, to determine the critical position and/or ultimate or critical serviceability load. This is an additional feature not present in previous elastic based analysis.

The spreadsheet contains all the calculations needed to analyse all aspects of a single span arch bridge. The spreadsheet has four parts: 'Main' is the input and output data sheet, 'Elements' undertake the calculations and in addition there is an advanced settings sheet and a result plot sheet.

Service or ultimate loads can be applied and results including deflections, stresses etc. obtained for all arch locations and load increments. This spreadsheet determines these

parameters directly and uses the inbuilt graphical functions within the spreadsheet to present these as a range of graphs. Furthermore, the critical failure/serviceability limit load can be obtained by the standard optimisation package within the spreadsheet which finds the optimum location by use of the simple micro code. This spreadsheet works for a range of fixed geometric shapes, segmental, elliptical etc. but can also analyse arches surveyed at a limited number of positions by using cubic spline interpolation. As well as the standard Castigliano analysis the spreadsheet also includes soil masonry interactions, multi-axles, thinning model, yielding, limited tensile strength and automatic axle load distribution. An example application is given at the end of the paper.

2   THEORY

The theory to be used in the current study is based on Castigliano(Castigliano 1879)s energy method. The energy method has previously been used in arches by Bridle and Hughes(Bridle R. J. 1990). The arch is divided into a number of elements and then Castigliano's theorem II is applied, where the component contributions are added numerically. This results in a 3x3 matrix, which is solved to determinate the abutment forces and moment. Castigliano's theorem II is then applied to evaluate the deflections of the individual elements. The soil pressures are determined assuming them to be linearly related to the horizontal component of the deflection. Pressures are initially assumed to be governed by at rest state and then, depending on the direction of movement, the pressures change linearly with displacement, towards the active or passive soil pressures, as appropriate. The load distribution methods combine two traditionally used methods based on an elastic half space and a predefined spread angle.

   The whole ensemble of arch ring elements is treated as a curved beam with each of the elements subjected to forces that can be resolved into axial, shear and moment components, illustrated in Fig.1.
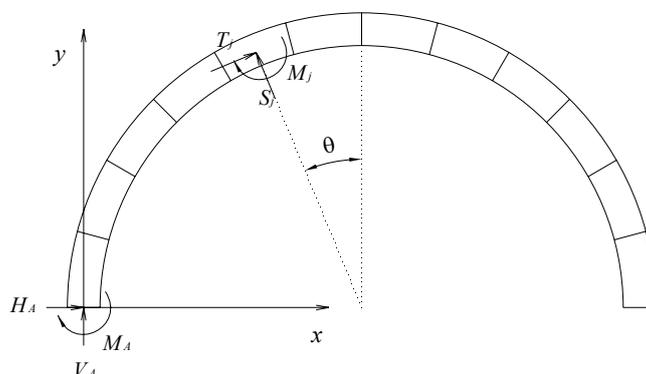

Figure 1 : The bridge arch's elements and their forces and moments

The integrations contained in the equations which follow are undertaken only from the left side abutment to any element j and then to the right side abutment.

   The total energy U resulting from the bending, axial and shear forces in the beam is written in equation (1).

$$U = \int \frac{M^2 ds}{2EI} + \int \frac{T^2 ds}{2EA} + k \int \frac{S^2 (1+v) ds}{EA} \qquad (1)$$

3 SPREADSHEET

3.1 Mainsheet layout

The spreadsheet containing all the calculations needed to analyse all aspects of a single span arch is shown in Fig.1. The spreadsheet is divided into 21 'boxes', A-U, these include data input boxes (including a surveyed data input option), calculation box, results and result charts. All data is in SI units. Table 1 contains the descriptions of the activities undertaken in each of these boxes.
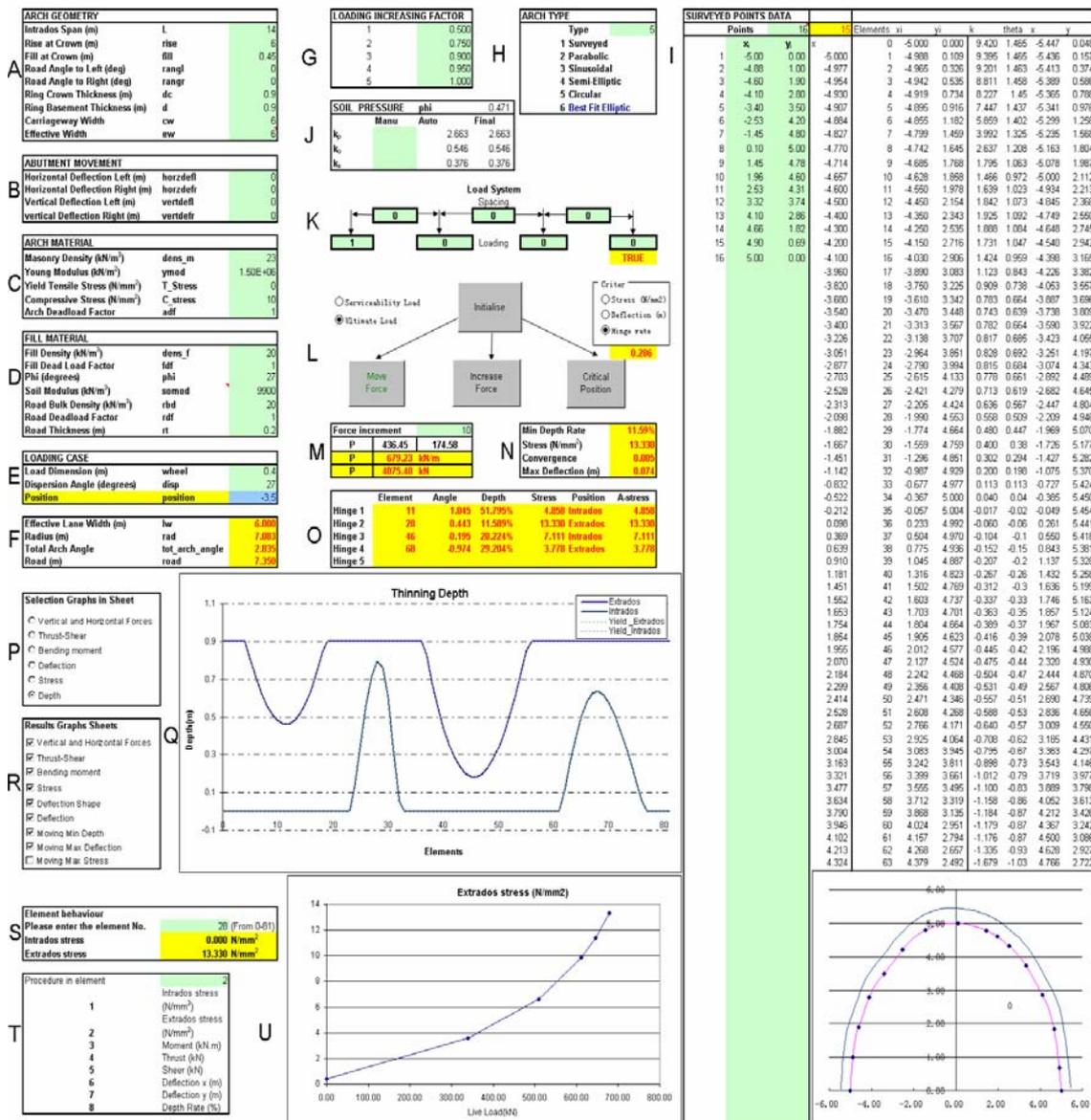

Figure 2 : Main sheet layout of Castigliano analysis spreadsheet

3.2 Macro and Solver

Box L contains the options to select ultimate limit load or serviceability limit load, and the associated serviceability criteria. Box L also contains the buttons to run the spreadsheet optimisation macro. The button 'Initial' runs a macro in the spreadsheet to set the initial data to enable the iteration process to start from a reasonable position, for example it positions the live load at the quarter point and sets the default live load at 40% of the MEXE PAL (Provisional Assessment Load) based on a 2.5 m lane width.   Later modifications to the arch being analysed

can be modelled from an existing converged solutions without the requirement to run "Initial" again.

Table 1 : Description of boxes in 'Main' sheet

Box	Description
A	Arch geometric input data
B	Abutment movement input data
C	Arch material input data
D	Fill material input data
E	Loading case input data
F	Simple calculation
G	Loading increasing factors input data
H	Arch type input data
I	Surveyed points input data
J	Coefficient of soil pressure input data
K	Loading pattern
L	Buttons to run critical position and limit (ultimate or serviceability) load using the
M	Limit/service load estimate and force result of the masonry arch bridge
N	Control value of the limit criteria
O	Hinge information of the arch ring
P	Graph options of results
Q	Selected result graph
R	Result sheets options
S	Element selection of its behaviour
T	Selected element's result graph options
U	Selected increasing load result graph

The 'Move Force' button runs the macro that moves the loading position, with the size of the steps being taken from the Moving Steps data in the Advanced worksheet.

'Increase Force' also uses the optimisation package solver in the spreadsheet., it constrains and optimises in the following way:-

(1) The Target is to maximise the highest stress/minimum ring effective depth over the bridge
(2) By changing the live load P
(3) Constraining that a converged solution (within specified tolerance) is obtained with either
All Stress < = Specified Compressive stress or
All Depth > = Specified Depth or
All Deflection <= Specified deflection.

The 'Critical Position' combines the two buttons 'Move Force' and 'Increase Force by Depth'.
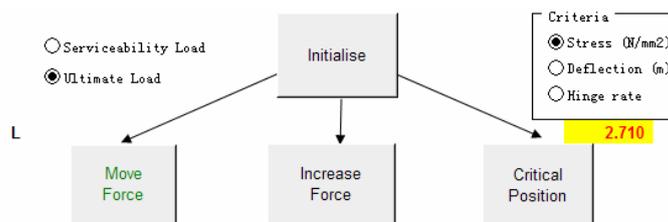

Figure 3 : Box L buttons to run the spreadsheet optimisation macro

3.3 Calculation sheet- 'Elements'

The bulk of the calculations are undertaken in the worksheet 'Elements'. The arch is automatically divided into 80 elements and 'Elements' undertakes all the integrations associated with the Castigliano analysis as well as determining the soil pressures, deflections etc. The data from 'Elements' is used to draw all the graphs and determines, and stores, solutions at each load iteration. The increments of load are applied in 5 steps, including the first unloaded step, giving in total 6 steps. Every step has at least 6 iterations to converge to the solution.

Fig.5 shows the calculations in a typical iteration for the first 9 elements only. The iteration uses the previous coordinates from the last iteration or step, the previous thickness of the arch, the calculation of the horizontal forces and vertical forces of every element and the solution of the (3x3) matrix derived by the theory for the left hand abutment forces. Then it uses the abutment moment, vertical force and horizontal force M V H to determinate the elemental vertical force, horizontal force and cross section mid point moment, thrust and shear forces. It then calculates the new ring thickness, arch coordinates and convergence of the cross section. The convergence is based on the changing depth of the arch ring. When the arch depth becomes stable, the load has converged. Finally, it calculates the deflection and modifies the horizontal soil forces towards the active or passive states. It then begins the next iteration.

3.4 Graphical displays

There are also two normal chart sheets and eight hidden chart sheets in the spreadsheet. The hidden sheets can be displayed from the excel menu or from the 'Q' box in the 'Main' sheet. These sheets are briefly introduced below.

The 'Arch View' sheet is the most useful graphical sheet. This sheet includes the original arch geometry, load pattern, abutment forces, thrust line and thinned shape. The 'Depth' sheet shows the original arch depth, thinning depth including yielding depth. The other eight sheets are hidden sheets:

(1) 'VHForces' sheet includes the vertical force and the horizontal force for every arch ring element,

(2) 'Thrust-Shear' sheet details the thrust and shear forces in every arch ring elements,

(3) 'Bending Moment' sheet details the moment in the ring centre after cracking and yield thinning,

(4) 'Stress' sheet shows the stress in the extrados and the intrados of arch ring elements. At the same time this sheet also shows the total depth of the arch, because the highest stress is always located in the thinnest part of the arch ring.

(5) 'Def. Shape' sheet shows the total deflected shape of the arch.

(6) The 'Deflection' sheet gives the separate x direction deflection and y direction deflection of the arch ring elements.

(7) 'Moving Min Depth' sheet displays the minimum depth determined over the entire arch ring element, when the load on the bridge is moved from one side to the other side.

(8) 'Moving Stress' sheet is the maximum stress determined in any arch ring element, when the load on the bridge is moved across the arch.

4 EXAMPLE

This example uses a quarter point loading to determine the ultimate load and uses a service load to find out the critical load position. This spreadsheet sets the default arch width as a 1 metre strip. The example's input data is listed in Table 2.

The 'Initial button' button in the 'Main' sheet was used first to set an initial load position and start load magnitude, then the 'Increase Force' button was used to obtain the quarter point ultimate load; the results are shown in Fig.5. The arch view chart shows the load pattern, load value, abutment forces, thrust line, original shape and the thinning shape.

	Angle	x	y	deffet	enert	slopl	dx	dx	dy	r	sigmay	vfdis	dx	dy	r	sigmay	vfdis	dx	dy	r	sigmay	vfdis
-1																						
0	1.417	-7.404	0.063	0.819	0.046	0.01	0.002	-3.945	7.281	8.281	0.000	0.000	-3.945	7.281	8.281	0.000	0.000	-3.945	7.281	8.281	0.000	0.000
1	1.400	-7.404	0.197	0.861	0.053	0.27	0.045	-3.923	7.149	8.155	0.000	0.000	-3.923	7.149	8.155	0.000	0.000	-3.923	7.149	8.155	0.000	0.000
2	1.364	-7.373	0.463	0.900	0.061	0.27	0.055	-3.873	6.887	7.902	0.000	0.000	-3.873	6.887	7.902	0.000	0.000	-3.873	6.887	7.902	0.000	0.000
3	1.329	-7.314	0.723	0.900	0.061	0.27	0.064	-3.814	6.627	7.646	0.000	0.000	-3.814	6.627	7.646	0.000	0.000	-3.814	6.627	7.646	0.000	0.000
4	1.293	-7.245	0.981	0.900	0.061	0.27	0.073	-3.745	6.369	7.389	0.000	0.000	-3.745	6.369	7.389	0.000	0.000	-3.745	6.369	7.389	0.000	0.000
5	1.258	-7.167	1.236	0.900	0.061	0.27	0.082	-3.667	6.114	7.129	0.000	0.000	-3.667	6.114	7.129	0.000	0.000	-3.667	6.114	7.129	0.000	0.000
6	1.222	-7.081	1.489	0.900	0.061	0.27	0.091	-3.581	5.861	6.869	0.000	0.000	-3.581	5.861	6.869	0.000	0.000	-3.581	5.861	6.869	0.000	0.000
7	1.187	-6.985	1.738	0.900	0.061	0.27	0.100	-3.485	5.612	6.606	0.000	0.000	-3.485	5.612	6.606	0.000	0.000	-3.485	5.612	6.606	0.000	0.000
8	1.152	-6.881	1.983	0.900	0.061	0.27	0.109	-3.381	5.367	6.343	0.000	0.000	-3.381	5.367	6.343	0.000	0.000	-3.381	5.367	6.343	0.000	0.000
9	1.116	-6.768	2.225	0.900	0.061	0.27	0.117	-3.268	5.125	6.078	0.000	0.000	-3.268	5.125	6.078	0.000	0.000	-3.268	5.125	6.078	0.000	0.000

vforc	hforc	veeef	heeef	halfv	halfh	at	ct	cee0b	cee1b	cee2b	ceeeb	a(1)	a(2)	a(3)	b(2)	b(3)	c(3)	d(1)	d(2)	d(3)
0.428	0.000	0.000	0.000	0.214	0.000	-0.081	0.000	0.000	0.000	0.000	0.000	0	0	0	0	0	0	0	0	0
11.959	20.313	6.193	10.156	5.980	10.156	-0.039	0.000	0.000	0.080	0.000	0.080	0	0	0	0	-1	5	1	0	0
12.969	19.383	18.657	30.005	6.484	9.692	0.000	0.000	0.000	0.000	5.770	5.851	0	0	0	1	-2	4	4	-10	26
13.869	18.447	32.076	48.920	6.934	9.223	0.000	0.000	0.000	0.000	11.823	17.674	0	0	0	2	-3	4	13	-50	78
14.660	17.508	46.340	66.897	7.330	8.754	0.000	0.000	0.000	0.000	17.673	35.346	0	-1	1	4	-4	4	32	-140	155
15.346	16.570	61.343	83.935	7.673	8.285	0.000	0.000	0.000	0.000	23.486	58.832	1	-1	1	6	-5	4	70	-300	258
15.927	15.636	76.980	100.038	7.963	7.818	0.000	0.000	0.000	0.000	29.261	88.093	1	-2	1	9	-6	4	136	-548	387
16.407	14.712	93.147	115.212	8.203	7.356	0.000	0.000	0.000	0.000	34.996	123.089	1	-3	2	12	-7	4	239	-901	541
16.789	13.799	109.745	129.468	8.394	6.899	0.000	0.000	0.000	0.000	40.684	163.773	1	-4	2	16	-8	4	390	-1376	720
17.076	12.901	126.677	142.818	8.538	6.451	0.000	0.000	0.000	0.000	46.321	210.094	2	-6	3	21	-10	4	601	-1990	923

Vforc	Hforc	M	T	S	eccentricity	Min_Str	Max_Str	a'	a	at	c	ct	stlen	deffet	deupp	xnew	ynew	convergence
803	147	-103	816	-23	-0.127	70.569	1924.016	-0.031	0.000	-0.050	0.000	0.000	0.850	0.850	0.475	-7.420	0.065	0.03668
797	158	-123	812	-19	-0.152	-52.581	1938.598	0.000	0.023	-0.061	0.000	0.000	0.839	0.839	0.481	-7.393	0.195	0.02712
785	177	-143	804	-13	-0.178	-165.774	1953.135	0.000	0.070	-0.070	0.000	0.000	0.830	0.830	0.485	-7.339	0.455	0.08488
771	196	-145	796	-6	-0.183	-193.558	1961.918	0.000	0.081	-0.081	0.000	0.000	0.819	0.819	0.490	-7.274	0.713	0.09866
757	214	-146	787	1	-0.186	-208.177	1956.335	0.000	0.087	-0.087	0.000	0.000	0.813	0.813	0.493	-7.203	0.969	0.10641
742	231	-145	777	8	-0.186	-209.446	1936.342	0.000	0.088	-0.088	0.000	0.000	0.812	0.812	0.494	-7.126	1.223	0.10817
726	247	-142	767	15	-0.185	-197.246	1901.959	0.000	0.085	-0.085	0.000	0.000	0.815	0.815	0.492	-7.041	1.474	0.10371
710	263	-137	757	22	-0.181	-171.505	1853.251	0.000	0.076	-0.076	0.000	0.000	0.824	0.824	0.488	-6.950	1.724	0.09254
694	277	-130	746	29	-0.174	-132.188	1790.320	0.000	0.062	-0.062	0.000	0.000	0.838	0.838	0.481	-6.853	1.971	0.07383
677	290	-121	735	36	-0.165	-79.290	1713.288	0.000	0.040	-0.040	0.000	0.000	0.860	0.860	0.470	-6.750	2.217	0.04628

Figure 4 : Typical iteration in spreadsheet

Table 2 : Example data

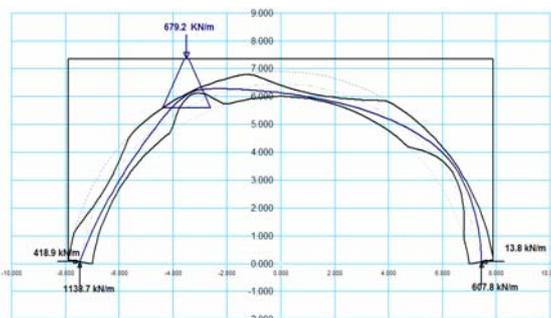|	
Intrados Span (m)	14
Rise at Crown (m)	6
Fill at Crown (m)	0.45
Ring Thickness (m)	0.9
Masonry Density (kN/m^3)	23
Young Modulus (kN/m^2)	1500000
Compressive Stress (N/mm^2)	10
Fill Density (kN/m^3)	20
Phi (degrees)	27
Soil Young Modulus (kN/m^3)	9900
Road Bulk Density (kN/m^3)	20
Road Thickness (m)	0.2
Load Dimension (m)	0.4
Dispersion Angle (degrees)	27
Arch type	circular

Figure 5 : Arch view of quarter point ultimate load result

The depth of the arch ring is displayed in exaggerated detail in Fig.6. This chart also includes the yield thickness. This chart gives very clear information about thinning and the effective arch ring thickness.
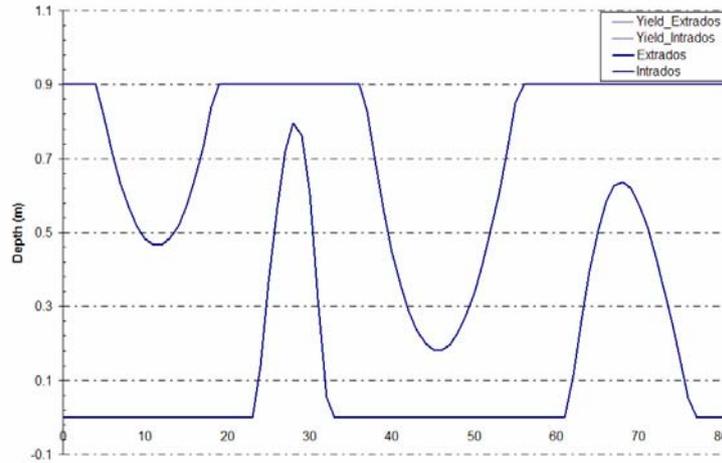

Figure 6 : Effective arch ring depth

The intrados and extrados stresses are showed in Fig.7. This chart also includes the effective depth of the arch ring in the background. This help to identify where these stresses occur.
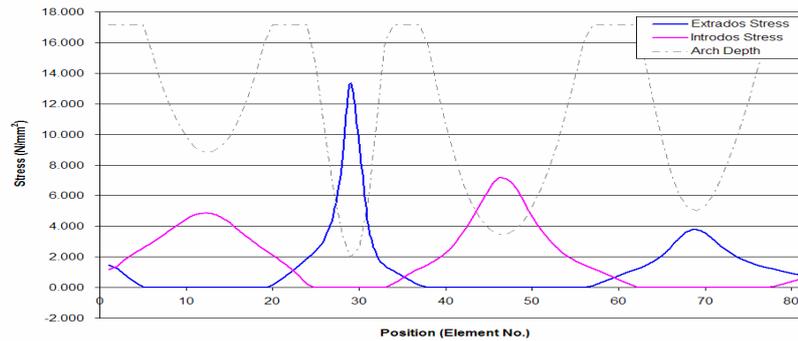

Figure 7 : Extrados and intrados stress

The use of the Move Load macro is displayed in Fig.8 which shows the variation of the minimum % of the arch ring depth for a moving live load. The minimum depth % is the ratio of the remaining elastic part of the element section to the overall initial section thickness.
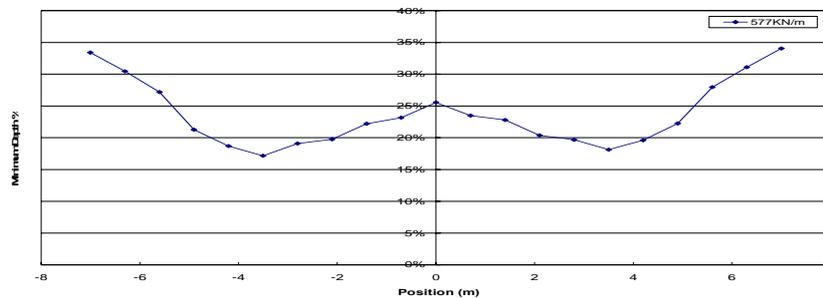

Figure 8 :   Minimum depth % for a moving load

5 CONCLUSIONS

A complex arch thinning Castigliano analysis has been presented and an example application of the method using a standard spreadsheet analysis has been demonstrated. The method uses simple geometric parameters to define the arch and standard optimisation software within the spreadsheet to determine critical loads and positions.

The current approach allows assessment engineers to fully understand, to intervene, to modify and to readily enhance their assessments. The spreadsheet is free to download (and modify) from http://masonry.engineering.cf.ac.uk/masonry-research/arch-spreadsheet.html .

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