ON OUT-OF-PLANE STABILITY OF BOW-STRING ARCH BRIDGES

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SUMMARY

Five arch bridge superstructures recently designed by the authors of the paper are presented in the paper. Spatial transformation models by using a FEM software were developed to analyse behaviour of the bridges. Verification of out-of-plane buckling of the arches is discussed in more detail. Comparison of using beam and shell finite elements for the mesh of main girder-arch load-carrying system is given. The variation of top bracing system and its influence on the stability of the arches is presented, as well. A minimum load amplifier to reach the elastic instability of arches is used to estimate the suitability of the bracings. Other parameters, which have significant influence on the stability, are discussed as well. Finally, comparison of the assessment using results produced by first and second order global analyses are presented.

Keywords: *Arch bridges, bow-string girder, buckling, out-of-plane stability of arches, initial imperfection, elastic critical buckling modes.*

1. INTRODUCTION

Arch bridges represent architecturally and structurally one of the most effective types of bridge structures. At present, they are the most commonly differentiated according to the position of the bridge-deck. Their pure, origin form may be seen in the “deck-arch” bridges (called as true arches as well), in which the deck is completely above the arch. They are particularly suitable for bridging large, deep valleys, providing sufficient space between vertical alignment of the transferred communication and the bottom of the valley for an optimal arch camber. Higher cost of production and installation of abutments, which require a very good foundation conditions for transmitting generally inclined reactions, is their disadvantage. On the other hand, the bridge deck does not limit design possibilities of the arches in terms of their number in the transverse direction as well as ensuring their out-of-plane stability. The second type represents so-called “bowstring-arch” or “tied-arch” bridges (in Central Europe known as Langer’s beam, as well), which are suitable for bridging wide watercourses or flat valleys. The bridge deck that is suspended on two side arches acts as a string transferring the horizontal component of the arc force, which results in lower requirements for the abutments. Their disadvantage is the limitation in relation to the number of arches in the transverse direction as well as ensuring their out-of-plane stability. The third type, known as "through-arch” bridge, combines the advantages and disadvantages of the two previous types. This paper is dealing with global analysis of the second type mentioned above, i.e.

the bowstring-arch bridges, especially with verification of out-of-plane buckling resistance of the arches. For this purpose, the spatial transformation FEM models of five arch bridge superstructures recently designed by the authors of the paper were utilised. The origin models were modified in the parametric study, focused on effectiveness of different ways of stiffening the arches against the out-of-plane buckling [1]. That parametric study has been extended in this paper by another bridge superstructure and by further FEM models using shell finite elements for the mesh of main girder-arch load-carrying system. The comparison of origin beam models with the shell models is made by the minimum load amplifier to reach the elastic instability of arches. Finally, comparison of the assessment using results produced by first and second order global analyses is presented.

2. DESCRIPTION OF BRIDGES

The more detailed description of the observed bridges has been already published [1, 2, 3], thus only brief preview of the main parameters of particular superstructures is presented here.

2.1. Railway Bridge over the Nosický Canal

The bridge, situated at km 159.038 of the railway line Bratislava – Zilina, is designed as a four-span two-line steel railway bridge with theoretical lengths of single spans 62.4 m + 124.8 m + 124.8 m + 62.4 m (Fig. 1).

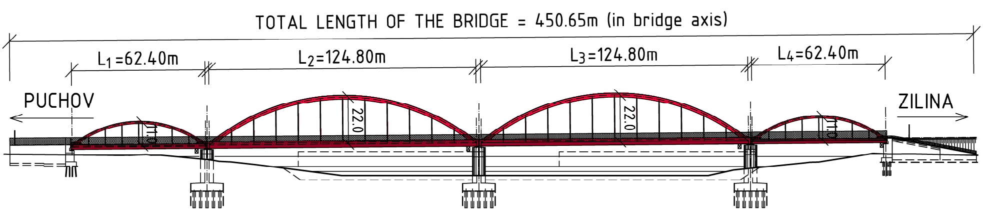

Fig. 1. *Longitudinal view of the bridge over the Nosicky Water Canal.*

All the steel superstructures consist of two bow-string girders with the bottom orthotropic bridge deck and the upper longitudinal bracing. The plate beams as well as the circular curved arches are designed from passable box-sections. The vertical hangers, designed from steel tubes filled with concrete, are hinged to the beams and arches. The more detailed description of the whole bridge can be found in [1, 2].

2.2. Railway Bridge over the Váh River in Trenčín City

The railway bridge is located immediately below the confluence of the Vah River and the Nosický Water Canal. The bridge is designed as a four-span structure under each railway line with theoretical lengths of single spans of 84.0 m (Fig. 2). The steel superstructures consist of two bow-string girders with the bottom orthotropic bridge deck and the upper longitudinal bracing. The plate beams are designed from opened unsymmetrical I-shaped cross sections, while the parabolic curved arches are made of closed box-sections. The vertical hangers are designed from the symmetrical I-shaped cross-sections, fixed to the beams and arches in the plane perpendicular to the plane of girders. The more detailed description of the whole bridge can be found in [1, 3].

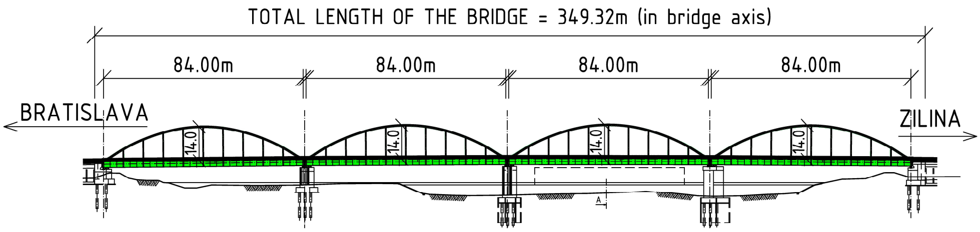

Fig. 2. *Longitudinal view of the bridge over the Nosický Water Canal.*

2.3. Railway Bridge over the Bela River in Liptovský Hrádok City

The third bridge is situated directly behind the railway station Liptovský Hrádok of the railway line Kosice – Žilina and it will cross the Bela River, a right-side inflow of the Váh River. The bridge is designed as two-line steel railway bridge with theoretical span length of 66.0 m (Fig. 3).

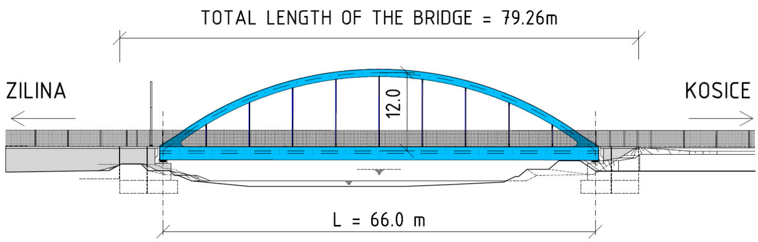

Fig. 3. *Longitudinal view of the bridge over the Bela River.*

The steel superstructure consists again of the bow-string girders with the bottom orthotropic bridge deck and the upper longitudinal bracing. The plate beams as well as the circular curved arches are designed from passable box-sections. The vertical hangers positioned in the tenths of span are made of circular hollow sections. The more detailed description of the whole bridge can be found in [1].

2.4. Road Bridge in Liptovský Hrádok City

The bridge on the road I/18 in Liptovský Hrádok City spans two local roads and five railway tracks (Fig. 4). The superstructure consists of two steel bow-string girders with theoretical span length of 81.0 m in axial distance of 12 m, connected with the bottom steel and concrete composite bridge deck and the upper longitudinal bracing. The plate beams are designed from opened symmetrical I-shaped cross sections, except from the edge parts, where the single-web cross-section changes to double-web closed cross-section for better connection of the arch box-section. The vertical hangers are designed from steel rods hinged to the girders and arches.

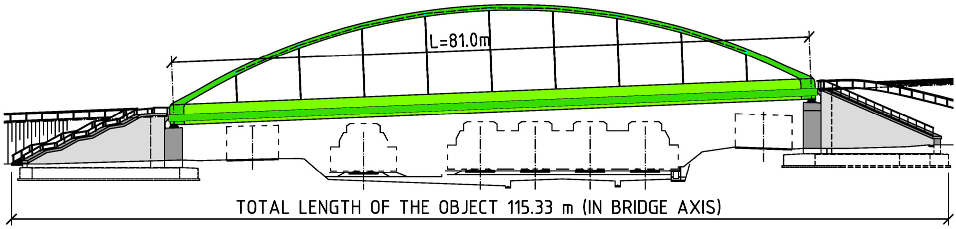

Fig. 4. *Longitudinal view of the road bridge in Liptovský Hrádok City.*

3. PARAMETRIC STUDY

3.1. Assumptions of the study

The parametric study, a part of which has been already published in [1], was primarily focused on influence of the upper longitudinal bracing on stability of the arches of four railway bridge superstructures. Different types of the bracing system (Fig. 5) were incorporated into origin computational models of the railway bridge superstructures. The first comparative model was considered without any top bracing system (I). Then, two basic types of top bracing systems were taken into account: the frame system (II) with varying number of cross-bars and the truss system (III) with various arrangements of diagonals. In all cases, the bracing members were designed of circular hollow sections. In case of truss bracing system the slenderness of members did not exceed the value of 150. In this paper, the study has been extended by the last aforementioned road bridge superstructure (Fig. 4). Another extension of the study was achieved using further comparative FEM models with shell finite elements for the mesh of main girder-arch load-carrying system. Simplified designation of the bridges from A to E for the purpose of presented study is given in following Tab. 1.

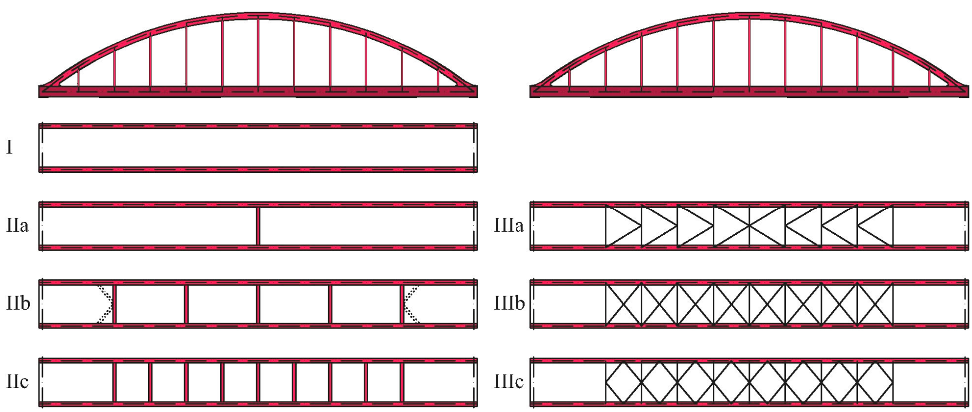

Fig. 5. *Considered types of upper longitudinal bracings.*

Table 1. *Basic parameters of bridge superstructures.*

Bridge designation	A	B	C	D	E
Arch/bridge span	62.4	66.0	81.0	84.0	124.8
Rise-to-span ratio	1/5.67	1/5.50	1/7.36	1/6.00	1/5.67
Traffic type	two-line railway	two-line railway	two-line road	single-line railway	two-line railway
Span-to-width ratio	5.07	5.28	6.80	13.44	9.90
Bridge deck	steel orthotropic	steel orthotropic	composite steel-concrete	steel orthotropic	steel orthotropic
Hangers	hinged tubes	hinged rods	hinged rods	fixed I-sections	hinged tubes
Main girder cross-section	box girder	box girder	plate girder	plate girder	box girder
Arch cross-section	box section	box section	box section	box section	box section
Arch-to-girder connection	rigid	rigid	rigid	rigid	rigid

3.2. FEM modelling

Utilization of a commercial software based on Finite Elements Method (FEM) allows for spatial behaviour of bridges. Steel plates of the orthotropic deck of railway bridge superstructures were meshed by shell finite elements, as well as the reinforced concrete deck of the road bridge. Two basic concepts for modelling of the rest bridge geometry were adopted. In so called „member" models, the beam finite elements were used for modelling main girders, arches, hangers and top bracings. The longitudinal and transversal stiffeners of the steel decks as well as the cross-beams of the steel-and-concrete composite deck were modelled as ribs of the shell members. Considering the actual structural details, all the arch-to-girder joints were considered as rigid. The connections of hangers were approximated by the hinge joints, except for hangers of welded I cross-section of the single track railway bridge D, where the joint was modelled as rigid in out-of-plane bending.

Unlike the previous models, in the "shell" models the main girders and arches, and also their relevant stiffeners and diaphragms, were meshed by the shell elements. The transversal stiffeners or cross beams as well as the frame bracing members and I-shaped hanger members were approximated by 2D elements too. The truss bracing members and also the hangers made of CHS profiles or rods were modelled by beam elements. Modelling of the other parts of bridge superstructures stayed unchanged. Attention was paid to correct modelling of joints to approximate real behaviour of structural details. The arch-to-girder connection including corresponding diaphragms in the case of analysed brides E is illustrated in Fig. 6.

Type of mesh elements used in FEM models are summarized in Tab. 2.

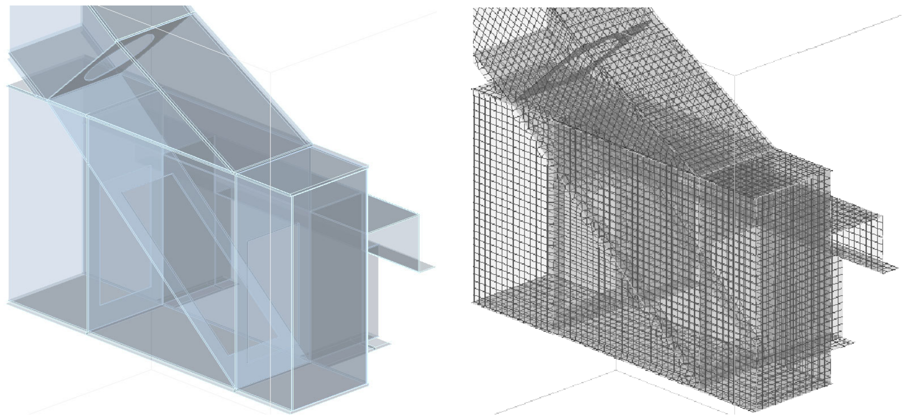

Fig. 6. *Detailed view on arch-to-girder connection of the bridge E in shell model alternative.*

Table 2. *Type of mesh elements used in FEM models.*

Use of element type in FEM models		**"MEMBER" models**		**"SHELL" models**		
		Railway bridges	**Road bridge**	**Railway bridges**		**Road bridge**
				single-line bridge	**two-line bridges**	
Applied for bridge		A, B, D, E	C	D	A, B, E	C
Deck parts	steel shells	shells	concrete shells	shells		concrete shells
	longitudinal stiffeners	beams (ribs)	-	beams (ribs)		-
	transversal stiffeners	beams (ribs)	-	shells		-
	cross beams	-	beams (ribs)	-		shells
Main carrying system	**main girders**	**beams**		**shells** (including stiffeners/diaphragms)		
	arches	**beams**		**shells** (including diaphragms)		
	hangers	beams		shells	beams	
Bracings	truss alternatives	beams		beams		
	frame alternatives	beams		shells		

3.3. Results of stability analyses

All the numerical models (beam models as well as shall models) were used for global stability analysis in order to get the first eigenmodes of the loss of structural stability, especially of the arches. These eigenmodes are quantified by the factor α_{cr}, by which the design load should be increased to cause the elastic instability in a global mode. The values α_{cr} are showed in Tab. 3, both for the beam models ($\alpha_{cr,b}$) and the shell models ($\alpha_{cr,s}$).

Table 3. *Calculated values of the factors α_{cr} for various bracing systems and FEM models.*

Bridge	A		B		C		D		E	
*	$\alpha_{cr,b}$	$\alpha_{cr,s}$	$\alpha_{cr,b}$	$\alpha_{cr,s}$	$\alpha_{cr,b}$	$\alpha_{cr,s}$	$\alpha_{cr,b}$	$\alpha_{cr,s}$	$\alpha_{cr,b}$	$\alpha_{cr,s}$
I	4.57	4.65	3.83	3.93	0.71	0.76	1.53	1.51	1.76	1.83
IIa	5.06	5.34	4.25	4.21	0.80	0.87	1.90	1.70	1.94	2.02
IIb	5.25	5.36	4.43	4.39	1.29	1.06	2.36	1.88	2.85	2.32
IIc	5.46	5.52	4.48	4.50	1.81	1.21	2.75	2.01	3.43	2.48
IIIa	12.61	13.40	8.82	9.37	5.72	6.20	3.04	3.21	5.69	6.41
IIIb	13.10	12.96	9.18	9.63	5.98	6.49	3.24	3.39	5.64	6.37
IIIc	13.34	12.31	8.80	9.15	7.13	7.32	3.39	3.53	5.71	6.37

* b - beam model, s - shell model

As could be expected, from the comparison of the amplifier values it can be stated that the frame bracing system is generally less effective than the truss one. In the case of narrow bridges with span-to-width ratio greater than 10.0, the frame system appears to be ineffective ($\alpha_{cr} < 3.0$). When using the truss bracing, the rhombic system seems to be the most effective. On the contrary, the K-truss system is the least effective, although only small differences were observed. If no bracing is provided, the out-of-plane stability of arches rapidly decreases with increasing span-to-width ratio, especially in the case of small fixation capability of main girder in arch-to-girder joint. In that case, insufficient stiffness of the opened girder cross-section in horizontal bending and torsion results in arch-ends rotation.

The study outlined a role of rigidity of the arch-to-girder connection. More precise modelling using shell elements is generally recommended. However, according to the study, application of shell elements does not necessary leads to more relevant stability data. Especially, in the case of slender cross-section, when many stiffeners are connected to thin plates, they have to be carefully taken into account in the structure model, including their connection. If local stability is not considered properly, it can result in increase of global deformations, e.g. the horizontal deformation of the arches in stability analysis.

In Fig. 7 the relations between $L_{cr,z}/L$ ratio and amplifier α_{cr} in the case of member models is shown. Actually, the out-of-plane buckling lengths of non-braced arches could be considered under 1/3 of the theoretical arch length for all analysed arches. When truss bracing is applied, the out-of-plane buckling length was under 1/5 of the arch length, regardless of the span and the truss type.

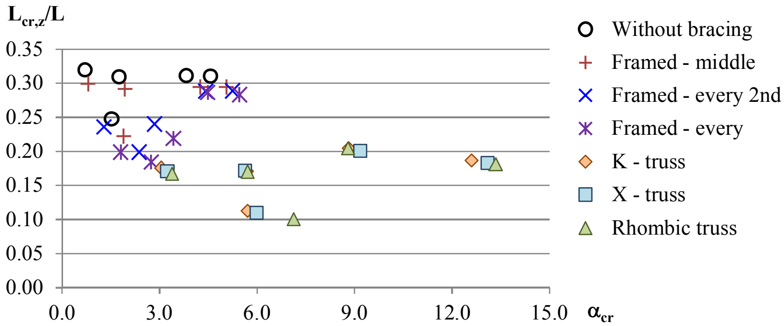

Fig. 7. *Relation between $L_{cr,z}/L$ ratio and amplifier α_{cr} in the case of member models.*

3.4. Verification of chosen arches

The obtained shapes of the elastic critical buckling modes (eigenmodes) from stability analyses were used for assessment of critical sections of arches by means of second order global analysis. The key problem is to estimate the amplitude of so-called unique global and local imperfection. In the case of common structures, especially when uniaxial bending and in-plane buckling are under consideration, the procedure given in [4, 5] can be utilized. When biaxial bending and torsion are present combined with more complicated geometry, a complicated iteration procedure should be executed. Moreover, most of common software still do not offer in second order analysis a possibility to solve separately internal forces induced by load cases combination and by the initial imperfections, respectively. Therefore, with regard to the aim of this study, following assumptions were adopted. The arch can be considered to be a member subjected to practically constant axial compression flexibly fixed against deflection at the both ends and also flexibly supported at the upper bracing nodes. The rigidity of the end supports depends on torsional and horizontal flexural stiffness of the end parts of main girder. For instance, in the case of box girder cross-section they may be thought to be almost perfectly rigid. Thus, the obtained global eigenmode of the loss of structural stability can be considered as the local initial out-of-plane bow imperfection of the arch member. Consequently, the amplitude of unique global and local imperfection η_{init} of the arch according to 5.3.2(11) in EN 1993-1-1 [6] may be supposed to be approximately equal to the local imperfection e_0 according to 5.3.2(3)b) in EN 1993-1-1 [6]. Thus, the value equal to L/250, which corresponds to the buckling curve "b", was applied to scale the shape of the elastic critical buckling mode. However, the length L is considered to be equal the out-of-plane buckling length of the arch L_{cr} that was calculated using the obtained critical factors α_{cr} from the stability analyses.

$$L_{cr,z} = \pi \sqrt{\frac{E \cdot I_z}{\alpha_{cr} \cdot N}} \tag{1}$$

Applying these amplitudes in the member models from the presented parametric study, the internal forces in critical sections were obtained using nonlinear second order global analysis. Consequently, the resistance of these critical cross-sections was evaluated and compared with the standard assessment with the equivalent column method according to 6.3.3 in EN 1993-1-1 [6]. The corresponding utilisation grades of the arch members obtained by first order analyses and second order analyses, respectively, are presented in Tab. 4. Only arches with no bracings, with the frame system in every hanger and with rhombic truss bracing, respectively, were chosen for verification (bracing systems designated as I, IIc and IIIc according to Fig. 5). From the comparison it is evident that the lover stability of arch is the higher differences can be observed. Bridges C and D without bracings were found to be unstable, therefore their utilisation grades are not given in the table.

Table 4. *Utilisation grades of chosen arch members.*

Bracing	Analysis	A	B	C	D	E
I	LA	0.762	0.800	-	-	1.242
	GNIA	0.754	0.740	-	-	0.899
IIc	LA	0.870	0.816	1.107	0.766	0.970
	GNIA	0.904	0.768	0.786	0.584	0.752
IIIc	LA	0.762	0.726	0.767	0.695	0.852
	GNIA	0.822	0.713	0.727	0.546	0.880

4. CONCLUSIONS

The paper presents study based on five bow-string arch bridge superstructures, recently designed by the authors of paper. Stability analyses using two types of spatial transformation models by means of FEM software were done. Based on the comparison of applying beam and shell finite elements, respectively, for the mesh of main load-carrying system, it can be stated that application of shell elements does not necessary leads to more relevant stability data. The rigidity of arch-to-girder connection plays also important role. If excessive local stability occurs in this joint, it can affect the global stability of arches.

Different types of bracings were analysed to evaluate their effectiveness. The study approved an assumption of higher effectiveness of truss bracing systems comparing to the frame bracings.

Finally, simplified method for applying initial geometric imperfections into the second order analysis is proposed. In this particular case, the out-of-plane buckling mode produced by stability analysis was considered as the local initial out-of-plane bow imperfection of the arch member. The utilisation grades from such arch verification are compared to the values obtained by the standard assessment with the equivalent column method.

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