Geometric analysis and load capacity of masonry arch bridges: a case study from Northwest Iberian Peninsula

Daniel V. Oliveira , Cláudia Lemos and Paulo B. Lourenço
ISISE, Department of Civil Engineering,University of Minho, Guimarães, Portugal

ABSTRACT: This paper reports the results from a geometrical survey carried out on 59 segmental masonry arch bridges from the adjacent geographical areas of Northern Portugal and Northwestern Spain. Departing from a detailed discussion of the geometrical results, eight reference bridges were defined as representative of the sample. Subsequently, the paper deals with a parametric assessment of the load carrying capacity of the reference bridges and discussion of results. It was found that the arch thickness and the physical properties of the fill are of paramount importance in terms of ultimate load capacity. Furthermore, the results indicate that the bridges from the sample are structurally safe with respect to the applicable legislation.

1 INTRODUCTION

The few works on masonry arch bridges carried out in Portugal have been basically centred in the survey of damage and strengthening solutions, while structural analysis and assessment are almost absent. Aiming at developing a methodology for a fast screening of structural safety of masonry arch bridges based on geometrical information, a research work was carried out, based on the following steps:

(1) geometrical survey of Portuguese and Spanish masonry arch bridges based on existing bibliography, with focus on the adjacent geographical areas of Northern Portugal and Northwestern Spain;

(2). definition of standard bridges geometrically representative of the sample;

(3) numerical assessment of the ultimate load carrying capacity of the standard bridges, including a parametric analysis.

The paper structure is as follows: the first part deals with the presentation and discussion of results from the geometrical survey and definition of the standard bridges. The second part of the paper is focused on the parametric numerical analysis and discussion of the most important parameters that control the ultimate load capacity of masonry arch bridges.

2 GEOMETRIC STUDY

2.1 Geometrical survey

A survey of the most important geometrical properties of ancient roadway masonry arch bridges was carried out based on available literature review (Nunes 1997, Carita 1998, Costa 2002, Fuentes 2005, IGESPAR 2008, Rodrigues 2008) and previous works (Luís and Santos 1999). In total 59 bridges from Portugal and Spain were considered, with emphasis on bridges located in the adjacent geographical areas of Northern Portugal (Minho and Trás-os-Montes provinces) and Northwestern Spain (province of Galicia). Indeed, 70% of the surveyed bridges are located in these three adjacent provinces. The predominance of these geographical areas is basically due to the existence and availability of data. As University of Minho is located in Minho province, visits to local bridges were possible and it was expected that the geographical vicinity might have led to the use of similar construction techniques (Brencich and Morbiducci 2007).

The 59 roadway bridges analyzed are constituted by segmental arches, either single or multi-span bridges, totalizing 207 spans. The non-geometrical parameters analyzed were the material used in the structure and the place and date of construction. As for the construction material used to build arches and piers, it was found that 79% of the bridges are made of granite. The other structural materials used are limestone, sandstone and schist. All bridges spanning more than 16 m are made of granite. This is an expected result as granite is the dominant rock in the Northern part of Portugal. In terms of location, 72% of the bridges from the sample are located in Portugal. This result is obviously conditioned by the availability and access to data. The date of construction is a very difficult parameter to assess, but based on the available information, 67% of the bridges were built up the 15th century.

The geometrical parameters collected were the number of spans, span *s*, rise *r*, crown thickness of arches *t* and the width *W* and height *H* of piers. Multi-span bridges are dominant within the sample, with 71% of the bridges. As for the relative depth, three classes were considered:

(1) shallow arch $0.00 < r/s \leq 0.25$
(2) semi-shallow arch $0.25 < r/s \leq 0.40$
(3) deep arch $0.40 < r/s$

In addition, bridges were also grouped into three categories as a function of their span, as follows:

(1) short span bridges $0.0 < s \leq 7.5$ (m)
(2) medium span bridges $7.5 < s \leq 15.0$ (m)
(3) large span bridges $15.0 < s$ (m)

The relation between the span, the rise and the rise to span ratio is illustrated in Fig.1 for all arches within the sample. The sample is dominated by arches up to 12m span. Most of the few large span bridges are located in Spain and were built during the Roman period. Deep and semi-shallow arch shapes are dominant, with a global average relative depth of about 0.40. However, rise to span ratios larger than 0.50 are not usual in segmental bridges and, most probably, are due to errors related to available geometrical data.

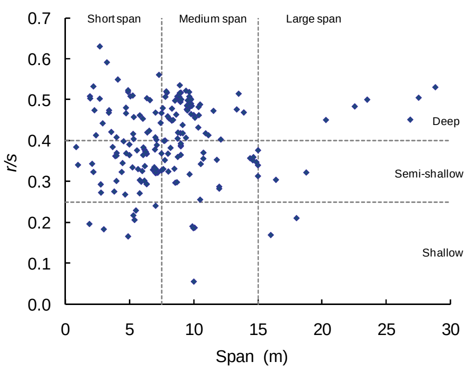

Figure 1 : Relation between the span and the rise to span ratio.

Short and medium span arches are the large majority of the arches surveyed, while less than 5% of the arches have a span larger than 15 m, see Figure. Moreover, most of the short and medium span arches present a relative depth higher than 0.25. This result means that the arches within the sample might be well represented by semi-shallow to deep arches with short to medium spans. Although not distinguished in the paper, the various geometrical ratios of Portuguese and Spanish bridges are quite similar (Lemos 2009), thus indicating that most probably the geographical proximity promoted the use of the same empirical design rules. The only difference concerns large span bridges, where all bridges with span larger than 20 m are located in Spain. Furthermore, the bridges built up the 15th century tend to exhibit higher thickness to span ratios than the less ancient bridges.

For the multi-span bridges (71% of the sample), piers were also included in the geometrical survey. However, the pier height is often a difficult parameter to characterize, due to lack of knowledge about the flow depth and the type of foundation. Figure represents the relation between the span and the width to span ratio. The span value used was computed as the average of adjacent span values. The figure shows that the width to span ratio is relatively constant for spans larger than 14 m, with an average value of 0.26. This value is 30% higher than the upper limit indicated by Campanela. On the other hand, the width to span ratio tends to increase with decreasing span, for spans less than 14 m. A qualitative trendline is illustrated in Figure . The existence of more piers in the river flow might have required the use of stockier piers for safety against floods and water stream.

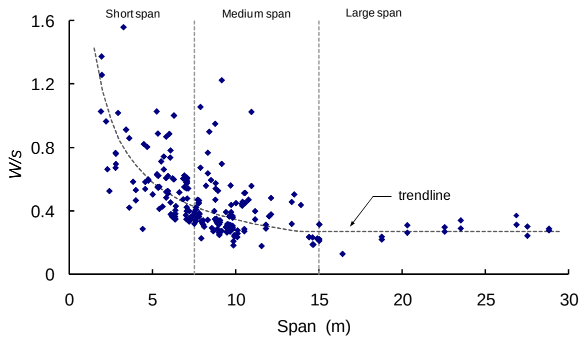

Figure 2 : Relation between the span and the width to span ratio.

2.2 *Reference bridges*

Based on the surveyed data, four single span reference bridges were defined as to be geometrically representative of the sample, as shown in Table **1**. Neither large span nor shallow bridges were considered due to their reduced presence in the sample as well as in the Northwest Iberian Peninsula. The analysis of the load capacity of reference bridges A, B, C and D will allow to provide a fast screening on the structural safety of the sample.

Table 1 : Single span reference bridges derived from the sample.

	semi-shallow arch	deep arch
short span	s = 5.0 m r/s = 0.30 t = 0.50 m (bridge A)	s = 5.0 m r/s = 0.50 t = 0.60 m (bridge B)
medium span	s = 10 m r/s = 0.30 t = 0.70 m (bridge C)	s = 10 m r/s = 0.50 t = 1.0 m (bridge D)

The thickness of the arches was defined according to the surveyed data, by considering representative values. For the other geometrical parameters, current values found in bridges from the Northwest Peninsular were assumed (Oliveira and Lourenço 2004a, Oliveira and Lourenço 2004b). The number of voussoirs was calculated assuming a usual thickness at intrados equal to 0.35m. In addition, a width of 4.0m was assumed for all the reference bridges, as well as a fill depth above the crown of about 0.40m.

3 ULTIMATE LOAD CARRYING CAPACITY OF SINGLE SPAN BRIDGES

3.1 *Numerical analysis*

Several methods and computational tools are currently available to simulate the structural response of masonry arch bridges. The most common idealizations of material behaviour are elastic behaviour, plastic behaviour and nonlinear behaviour. For a detailed discussion the reader is referred to Lourenço. Among the available computational methods proposed in literature to evaluate the load carrying capacity of masonry arch bridges, the rigid block computational limit analysis method is the most generally applicable (Gilbert and Melbourne 1994, Gilbert and Ahmed 2004). The applicability of limit analysis to masonry structures modelled as assemblages of rigid blocks connected through joints depends on few basic hypotheses, which are usually acceptable for the case of stone arch bridges (Orduña and Lourenço 2003).

In this study the computer software Ring1.5 (Gilbert and Ahmed 2004, Gilbert 2005), designed to compute the ultimate load carrying capacity of masonry arch bridges, was used. This software is based on the rigid block limit analysis method. Within it, single or multi-span bridges are modelled as in-plane structures, where arches are modelled as an assemblage of rigid blocks. The collapse state (collapse load and collapse mechanism) is computed through the use of limit analysis. For further details about the software, the reader is referred to Gilbert and Ahmed, Gilbert and Gilbert.

Input data within Ring 1.5 is composed by the bridge geometry, loading and material properties of the arches and fill. The geometry of the reference bridges has been fully defined based on the geometrical survey and on current values found in similar bridges. Besides the self-weight of the materials (masonry and fill), a live load composed by the standard Portuguese vehicle (RSA 1983) was considered. This standard vehicle is composed by three axles equally spaced by 1.5m, with a load of 100 kN per axle. With regard to material properties and in the absence of comprehensive in-situ test results, those were considered to assume typical values found in similar surveyed bridges (Oliveira and Lourenço 2004a, Oliveira and Lourenço 2004b), see Table **2**. In particular, the load dispersion through the fill was modelled according to the classical Boussinesq distribution, with a dispersion angle of 30º, and a earth pressure coefficient k_p based on the Rankine theory and equal to half of the value adopted for arches was used (Smith et al. 2004).

Table 2 : Material and mechanical properties adopted for the reference bridges.

Stone masonry	Fill material
self-weight = 25 kN/m3	self-weight = 20 kN/m3
friction angle = 30º	fill friction angle = 30º
compressive strength = 5 MPa	fill-barrel friction angle = 20º

The ultimate load carrying capacity is expressed in terms of a load factor, being a different load factor associated to each possible location of the moving vehicle, thus meaning that the minimum value of all possible load factors is the only one of interest.

3.2 Parametric analysis

In order to get a deep insight of the most important parameters that control the load capacity of bridges, a parametric analysis was performed on each of the reference bridges. The variables that most influence the collapse load were identified by means of a previous general parametric analysis (Lemos 2009). Therefore, the relevant variables considered here are geometrical and mechanical parameters of the arch and geometrical, mechanical and physical parameters of the fill, as follows:

(1) arch thickness (t);
(2) compressive strength of masonry (f_c);
(3) fill depth at the crown (h);
(4) mobilized earth pressure coefficient (k_p);
(5) physical properties of the fill (γ).

The physical properties of the fill, here represented by the symbol γ, encompass its self-weight and internal friction angle. The variation of the physical properties of the fill implies directly the simultaneous and coherent variation of both parameters, as well as indirectly the variation of the earth pressure coefficient and of the fill-barrel friction angle (Smith et al. 2004). The values adopted for the parametric analysis are provided in Table **3**. Besides the reference values, marked in bold, four additional physically significant values were considered for each of the parameters listed above. Each cell represents an independent numerical analysis, as only one parameter was varied in each run.

3.3 Discussion of results

Table **4** shows the load factors found for the four reference bridges. These factors were computed using the reference parameter values. The load factor is higher than 7 in all cases, which seems to indicate that reference bridges are structurally safe.

Table 3 : Values adopted for the parametric analysis (reference values marked in bold).

	Parameter		Unit	Parametric variation				
Fill	fill properties (γ)		(°; kN/m3)	(20; 18)	(25; 19)	(30; 20)	(35; 21)	(40; 22)
	fill depth (h)		[m]	0.0	0.2	0.4	0.6	0.8
	mobilized earth pressure coefficient (kp)		-	0.30	0.40	0.50	0.75	1.00
Arch	compressive strength (fc)		[MPa]	3	4	5	10	20
	arch thickness (t)	A	[m]	0.35	0.42	0.50	0.58	0.65
		B		0.40	0.50	0.60	0.70	0.80
		C		0.50	0.60	0.70	0.80	0.90
		D		0.70	0.85	1.00	1.15	1.30
Piers	width of piers (W)	A	[m]	1.75	2.5	3.25	4.00	4.75
		B		1.75	2.5	3.25	4.00	4.75
		C		2.00	3.00	4.00	5.00	6.00
		D		2.00	3.00	4.00	5.00	6.00
	height of piers (H)	A	[m]	7.0	10.0	13.0	16.0	19.0
		B		7.0	10.0	13.0	16.0	19.0
		C		9.0	12.0	15.0	18.0	21.0
		D		9.0	12.0	15.0	18.0	21.0

Table 4 : Load factor for the four single span reference bridges considered.

	semi-shallow arch	deep arch
short span	10.6	7.5
medium span	8.3	9.5

The results from the parametric analysis are summarized in Figure1, where the load factor (denoted as Fc) is related with the five parameters under analysis, for each reference bridge. The curves were normalized by with respect to results and data of reference bridges. This means that the point with coordinates (1;1) corresponds to the response of a reference bridge with reference data. The increase of any of the parameters causes the increase of the load factor for all the four bridges, but not in the same way. The variation of the arch thickness is very important to all bridges. While the variation of the mobilized earth pressure coefficient and of the physical properties of the fill affect more the deep bridges, due to the soil stabilization effect, the variation of the fill depth affects more short span bridges. With regard to the compressive strength, its variation effect is important mainly for very low compressive strength values and shallow arches. For these cases, it is important to know the compressive strength as to analyse the possibility of fragile failures, prior to the development of a ductile mechanism.

Figure1 shows beyond any doubt that the arch thickness and the physical properties of the soil are the most influential parameters of the load factor value. This result is of major importance as the values assumed in Table **3** for these two parameters are likely to be found in reality. The importance of the other three parameters depends on the bridge type. The mobilized earth pressure coefficient is most important for deep arches (B and D) due to the available fill depth and its stabilizing effect. For semi-shallow arches, the fill depth is more important for short span bridges (A), while all the three parameters have similar importance for medium span bridges (C).

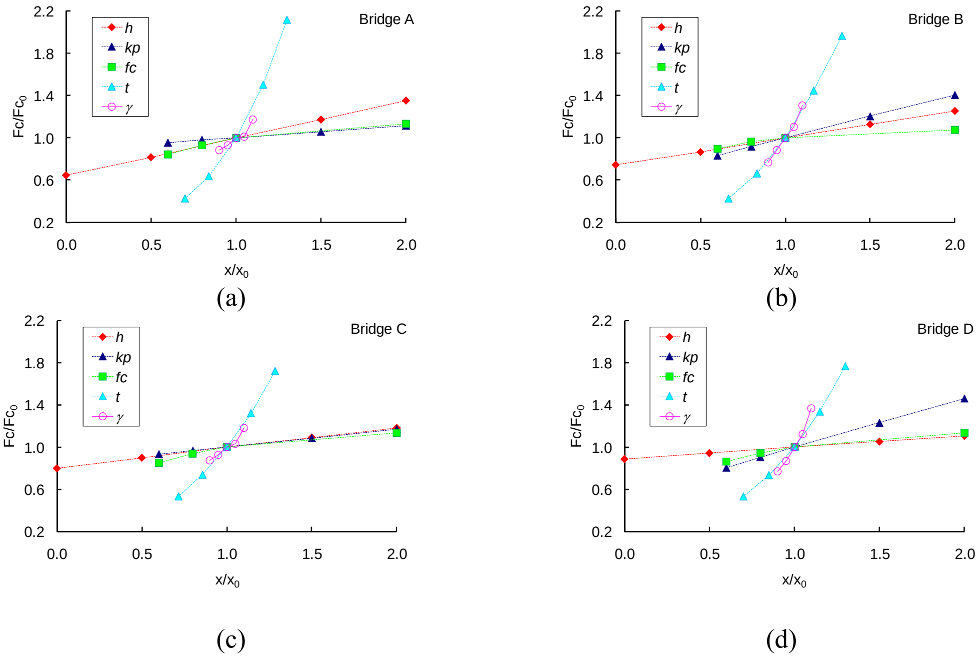

Figure1:Non-dimensional relationship between the load factor and the 5 parameters considered for: (a)semi-shallow short span bridge (type A); (b)deep short span bridge (type B); (c)semi-shallow medium span bridge (type C); (d)deep medium span bridge (type D). The variable *x* is referred to any of the five parameters considered.

4 ULTIMATE LOAD CARRYING CAPACITY OF MULTI-SPAN BRIDGES

Multi-span masonry arch bridges are often analysed as a series of separate single spans due to the presence of robust piers and the use of simpler numerical models. However, the ultimate load of a multi-span bridge modelled as such can be significantly lower than the value computed by omitting adjacent spans.

4.1 Numerical analysis

Aiming at assessing the load factor of multi-span masonry arch bridges, four multi-span reference bridges are considered next. Each of these reference bridges is composed of five equal spans. The arches of these additional four multi-span bridges have the same characteristics as the reference single span ones, see section2.2. The geometry of the piers was based on the geometrical survey, discussed in section2.1. Width to span ratios of 0.50 and 0.30 were adopted for short span and medium span bridges, respectively. As for the height to width ratio, a value of 4 was considered for all piers, see also Table **3**.

Multi-span bridges were modelled again resorting to Ring1.5 software, exactly in the same way as single span bridges did. The number of pier blocks was not available in literature, being adopted blocks with an approximate height of 0.60m, which is an usual value found in bridges from Minho region (Oliveira and Lourenço 2004a, Oliveira and Lourenço 2004b). Moreover, the influence of the number of blocks on the variation of the load factor, evaluated within a preliminary parametric analysis (Lemos 2009), was found to be low.

4.2 Parametric analysis

To characterize the sensibility of the ultimate load to the geometry of piers, four physically significant values were considered for piers width and height, in addition to the reference value, as illustrated in Table 3. Within this section, only the geometry of piers was varied. It is expected that the characterization of how the height of piers influences the load capacity might allow understanding the importance of errors associated to the measurement of this geometrical parameter.

4.3 Discussion of results

Table illustrates the load factor for the four multi-span reference bridges, computed using the reference parameter values. For all bridges, the load factor is higher than7, indicating that bridges are structurally safe. By comparing Table 4 with Table 5, it was found that multi-span bridges composed of deep arches (bridges B and D) modelled as such or as a series of separate single spans exhibit the same ultimate capacity. This is due to the occurrence of local collapse mechanisms (failure of a single arch). However, it is possible to identify a visible decrease of load factors associated to semi-shallow arch bridges (bridges A and C), namely 23% for short span and 11% for medium span bridges. Here, failure involves adjacent spans (global collapse mechanism). These results show that the computation of the load factor in multi-span masonry arch bridges using a single span is adequate only if a local failure mode is present.

Table 5 : Load factor for the four multi-span reference bridges considered.

	semi-shallow arch	deep arch
short span	8.11	7.54
medium span	7.33	9.51

The results from the parametric analysis are presented in Figure , where the load factor is related with the pier width and height. Within the normalization performed, the results obtained using reference parameter values are represented by the point with coordinates (1;1), as above. The increase of pier width causes the increase of the load factor up to a given threshold, defined as the shift of global to local collapse mechanisms, see Figure (a). Further increases of pier width are characterized by the failure of a single arch. Once arches were considered equal within a given reference multi-span bridge, the load factor cannot increase any longer. As for the variation of pier height, Figure (b) shows that its increase originates a decrease of the load factor, but only from a given threshold value onwards. This pattern, also registered in Figure (a), is fully visible for bridge D only. Within a given bridge, all local collapse mechanisms present the same load factor. In opposition, the load factor associated to global collapse mechanisms decreases with the decrease of pier width or increase of pier height. A threshold delimits the transition between local and global collapse mechanisms.

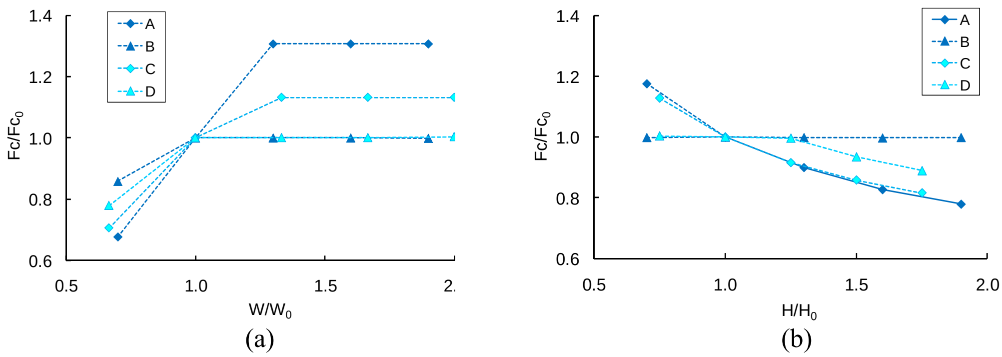

Figure 4 : Non-dimensional relationship between the load factor and: (a)width of piers, (b)height of piers. A-D indicate the reference bridge being considered

5 CONCLUSIONS

A sample of 59 roadway bridges mainly located in the adjacent geographical areas of Northern Portugal and Northwestern Spain was considered for a parametric study. The sample is dominated by arches up to 10m span, typically with a relative depth higher than 0.25, where 67% of the bridges were built up the 15th century. The geometrical ratios of Portuguese and

Spanish bridges analyzed are rather similar, thus indicating that most probably the geographical vicinity promoted the use of the same construction techniques and practical rules.

The numerical results showed that the arch thickness and the physical properties of the soil are the most influential parameters for the load factor for single span bridges. However, the estimation of the arch thickness is sometimes problematic because the external (visible) arch thickness might differ from the internal (effective) value. As for the multi-span bridges, it was found that all local collapse mechanisms present the same load factor, whereas for global collapse mechanisms the load factor decreases with the decrease of pier width or increase of pier height.

For single and multi-span reference bridges, the load factor is always higher than 7, so in global terms the major part of bridges within the sample seem to be structurally safe with respect to the applicable legislation. However, due to lack of maintenance, many bridges show damage, which might reduce its ultimate load carrying capacity. A way to incorporate such features in the numerical analysis is by considering suitable reductions of key parameters, as the arch thickness, the arch width or the compressive strength of masonry.

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