Arch Bridges
ARCH´04
P. Roca and E. Oñate (Eds)

CONCRETE-FILLED STEEL TUBULAR ARCH BRIDGE: DYNAMIC TESTING AND FE MODEL UPDATING

Wei-Xin Ren*, Bijaya Jaishi* and Zhou-Hong Zong*

* Bridge Stability and Dynamics Lab.
Department of Civil Engineering
Fuzhou University
Fuzhou, Fujian Province 350002, P.R. of China
e-mail: ren@fzu.edu.cn, web page: http://bridge.fzu.edu.cn/

Key words: Arch, Bridges, Concrete-filled steel tubular arch, Dynamic test, Modal analysis, Modal parameter identification, Finite element method, Model updating

Abstract. *This paper presents the experimental modal analysis, analytical modal analysis and finite element model updating of a half-through concrete-filled steel tubular (CFST) arch bridge. Field dynamic test was carried out just prior to the official opening of bridge under ambient vibration excitations. Two independent but complementary output-only modal identification techniques were used for modal identification. They were the peak-picking (PP) method in the frequency domain and the stochastic subspace identification (SSI) method in the time domain. The 3-D finite element model was developed and performed analytical modal analysis to achieve natural frequencies and mode shapes. A practical and handy finite element model updating method is then presented using the field ambient vibration test results. The objective function considers the residuals of frequencies, Modal Assurance Criterion (MAC), as well as flexibility. An eigenvalue sensitivity study is carried out to see the sensitive parameters to concerned modes. The objective function is minimized using the least square algorithm. The updated finite element model is able to produce natural frequencies in close agreement with the experiment results with enough improvement on MAC value of concerned modes still preserving the physical meaning of parameters. The updated finite element model can serve as the baseline for the long-term health monitoring and damage detection of the bridge.*

1 INTRODUCTION

The arches are mainly subjected to compression. The concrete-filled steel tubular (CFST) arches, taking advantages of both steel and concrete, make the arch spans longer. The first CFST arch bridge in China was completed in 1990. After that, more than two hundred bridges have been built or under construction in China. The span length has reached to 460m or even longer. The nonlinear static behavior of CFST arch bridges was intensively studied by experiments and finite element analysis[i]. However, the dynamic characteristics of this type bridges have been studied rarely in the literatures.

The dynamic properties of bridges can be obtained by either experimental modal analysis or analytical modal analysis. The modal parameters such as natural frequencies, damping ratios and mode shapes can be identified through the field vibration measurements. Finite element (FE) method is now a common way to the analytical modal analysis where the dynamic characteristics of bridges can be calculated.

In modern analysis of bridge dynamics, much effort is devoted to the derivation of accurate models. These accurate models are used in many applications like damage detection, health monitoring, structural control, load-carrying capacity evaluation. The FE model of a bridge is usually constructed on the basis of highly idealized engineering blue prints and designs. When field tests are performed to validate the analytical model, inevitably their results, commonly natural frequencies and mode shapes, do not coincide with the expected results from the theoretical model. The purpose of FE model updating is to modify the mass, stiffness and damping parameters of the numerical model in order to obtain better agreement between numerical results and test data.

A number of dynamics based model updating methods have been proposed[ii,iii,iv] The sensitivity-based parameter updating approach has an advantage of identifying parameters that can directly affect the dynamic characteristics of the structure. The objective function is often built up by the residual between the measurement results and the numerical prediction. There are commonly three expressions mostly used for this purpose, which are frequency residual, mode shape considered function and flexibility residual. Most of sensitivity-based approaches only consider the eigenvalue residual.

The objective of this paper is to present the experimental and analytical dynamic analysis on a newly constructed concrete-filled steel tubular arch bridge in Xining, China. Just before the bridge opening to traffic, the field ambient vibration test was performed and bridge dynamic characteristics were identified. Three-dimensional finite element model of the bridge was developed based on the original blue prints. An eigenvalue sensitivity study is then carried out to see the effect of various parameters to concerned modes, according to which the most sensitive parameters are selected for updating. The objective function, consisting of eigenvalue residual, MAC consideration function and flexibility residual, is minimized using the least square algorithm. The updated finite element model is able to produce natural frequencies in close agreement with the experiment results still preserving the physical meaning of parameters. It is demonstrated that finite element and experimental modal analysis provide a comprehensive investigation on the dynamic properties of the bridge. The analytical modal analysis through three-dimensional finite element modeling gives a detailed description of the physical and modal characteristics of the bridge, while the experimental modal analysis through the field dynamic tests provides a valuable source of information to validate the drawing-based idealized finite element model.

2 BRIDGE DESCRIPTION

The Beichuan River Bridge with a span of 90m, as shown in Figure 1, is a half-through concrete-filled steel tube tied arch bridge over the Beichuan River located at the center of Xining City, Qinghai Province, China. The superstructure of the bridge consists of the vertical load bearing system, the lateral bracing configuration, and the floor system. The cross-section of two main arch ribs is a truss of four concrete-filled tubes, with the dimension of 650×10mm. The rib height is 3000mm. There are 32 main suspenders of steel wire ropes that are vertically attached on main arch rib and floor system is suspended through it. Each of these 32 main ropes consists of 127 smaller bars each with a diameter of 5.5 mm. The floor system consists of a 250 mm thick concrete slab supported directly by cross girders at a spacing of 5m. The typical rectangular cross section of the cross girder is 0.36×1.361 m. The length of each cross girder is 21.6 m between the suspenders. The main arch ribs are fixed at two abutments, and connected by 4 pre-stressed strands each side in the longitudinal direction which acts as tied bars. Each stands were prestressed by 2200kN force. The concrete deck is supported by the expansion bearings at the two ends.

Figure 1: The Beichuan River CFST arch bridge

3 FIELD AMBIENT VIBRATION TESTS

The dynamic testing of a structure provides a direct way to obtain the bridge dynamic characteristics. Compared with traditional forced vibration testing, the ambient vibration testing using natural or environmental vibrations induced by traffic, winds and pedestrians is more challenge to the dynamic testing of bridges. Ambient vibration tests have an advantage of being inexpensive since no equipment is needed to excite the bridge. It corresponds to the real operating condition. The service state need not to be interrupted to use this technique.

Just prior to officially opening the bridge, the field ambient vibration tests on the Beichuan River arch bridge were carried out. The equipment used for the tests included accelerometers, signal cables, and a 32-channel data acquisition system with signal amplifier and conditioner. Accelerometers convert the ambient vibration responses into electrical signals. Cables are used to transmit these signals from sensors to the signal conditioner. Signal conditioner unit is used to improve the quality of the signals by removing undesired frequency contents (filtering) and amplifying the signals. The amplified and filtered analog signals are converted to digital data using an analog to digital (A/D) converter. The signals converted to digital form are stored on the hard disk of the data acquisition computer.

Measurement points were chosen to both sides of the bridge at a location near the joint of suspenders and deck. As a result, a total of 32 locations (16 points per side) were selected and measured. The measurement station arrangements are shown in Figure 2. Four test setups were conceived to cover the planned testing area of the bridge. One reference locations was selected near each side of abutment for each setup. The sampling frequency on site for vertical data and transverse data 80Hz and 200 Hz respectively with a recording time of 15-20 minutes. The force-balance accelerometers were directly mounted on the surface of the bridge deck.

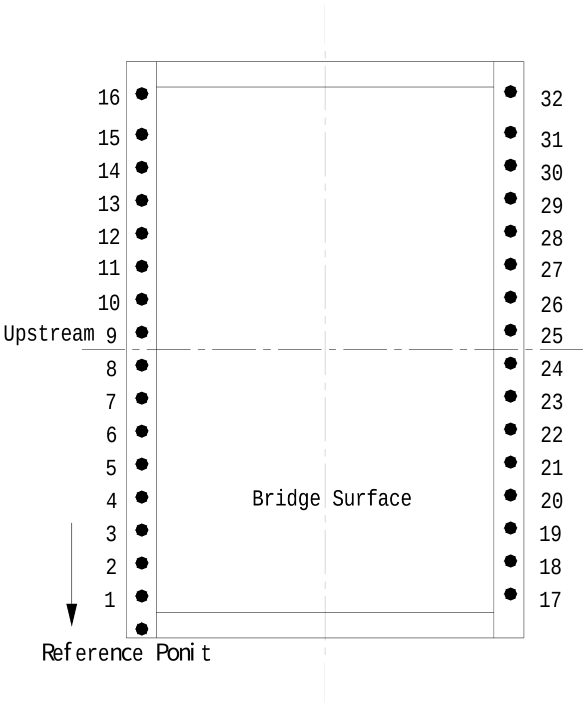

Figure 2: Measurement stations

Figure 3: Accelerometers mounted on bridge deck

Ambient vibration measurements do not lend to frequency response function or impulse response function calculations because the input excitations are not measured. The modal parameter identification is therefore based on the output-only data. Two complementary modal parameter identification techniques are implemented here. They are rather simple peak picking (PP) method in frequency-domain and more advanced stochastic subspace identification (SSI) method in time-domain. The data processing and modal parameter identification were carried out by MACEC[v]. The theoretical background of both identification techniques can be referred to Peeters[vi] as well as Van Overschee and De Moor[vii].

Mode | FE analysis | Stochastic subspace identification

1st vertical

2nd vertical

3rd vertical

4th vertical

1st torsion

1st transverse

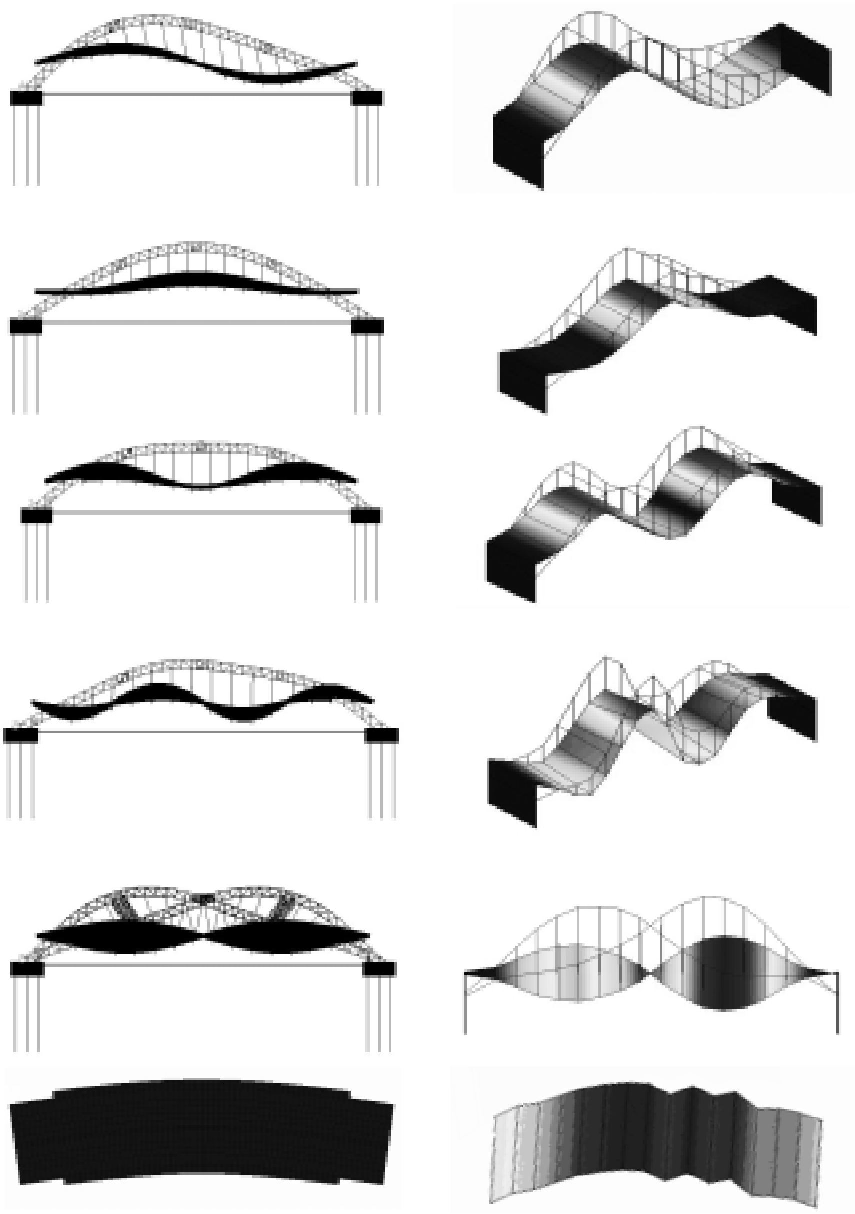

Figure 4: Identified and calculated mode shapes of the bridge

The identified frequencies and damping ratios from field ambient vibration measurements are summarized in Table1. It can be observed that the identified frequencies have a good agreement between the peak picking in frequency domain and the stochastic subspace identification in time domain. The typical identified mode shapes from the stochastic subspace identification are shown in Figure 3.

Table 1 : Identified frequencies and damping ratios

Mode	Peak-picking	Stochastic subspace identification	
	Frequency (Hz)	Frequency（Hz）	Damping Ratio（%）
1st vertical	2.012	2.002	0.80
2nd vertical	2.519	2.511	2.40
3rd vertical	3.457	3.473	1.20
4th vertical	4.628	4.624	1.30
1st torsion	2.812	2.827	1.00
2nd torsion	3.926	3.864	1.90
3rd torsion	5.390	5.419	1.50
1st transverse	2.776	2.780	1.20

4 FINITE ELEMENT MODELING OF THE BRIDGE

The three-dimensional linear elastic finite element model of the bridge was constructed. The arch member, cross girder, and bracing members were modeled by two-node beam elements. All suspenders were modeled by the truss elements. The surface floor was modeled as shell elements. As a result, 3120 nodes, 3446 elements and 14060 active degrees of freedoms were recognized on the model as shown in Figure 5. The calculated mode shapes are compared with those identified from ambient vibration measurements in Figure 4.

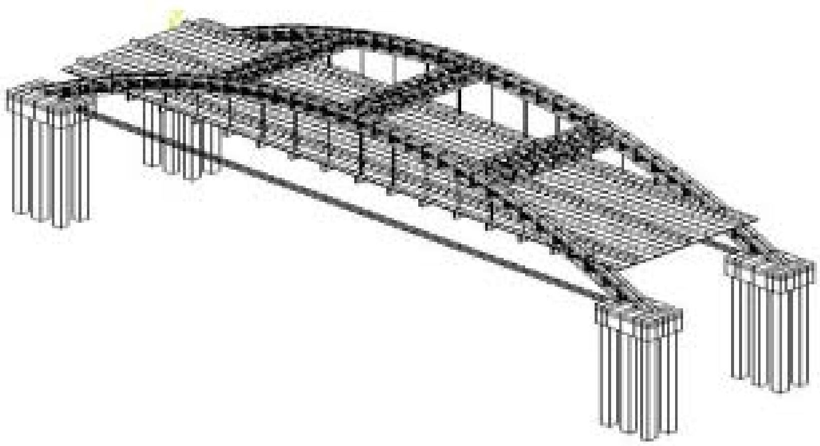

Figure 5: Three-dimensional finite element model of the tested bridge

5 FINITE ELEMENT MODEL UPDATING

5.1 Theoretical background

An objective function Π reflects the deviation between the finite element results, for a constant mesh density and the real behaviour. The model updating may be posed as a minimization problem to find x^* design set such that

$$\Pi\left(x^*\right) \le \Pi(x), \qquad \forall x \tag{1}$$

The general objective function formulated in terms of the discrepancy between finite element and experimental eigenvalues and mode shapes is shown below respectively.

$$\Pi_1(x) = \sum_{i=1}^{m} \alpha_i \left(\frac{\lambda_{ai} - \lambda_{ei}}{\lambda_{ei}} \right)^2, \quad 0 \le \alpha_i \le 1 \tag{2}$$

$$\Pi_2(x) = \sum_{i=1}^{m} \beta_i f_i \;, \quad 0 \le \beta_i \le 1 \tag{3}$$

where α_i and β_i are weight factor to impose a relative difference between eigenvalue and mode shape deviations respectively because these entities may have been measured with different accuracy. λ_{ai} and λ_{ei} are the finite element and experimental eigenvalue of the i*th* mode respectively. f_i is the mode shape related residual. After trying several expressions, Moller and Friberg [viii] proposed the following expression

$$f_i = f\left(MAC_i\right) = \frac{\left(1 - \sqrt{MAC_i}\right)^2}{MAC_i} \tag{4}$$

in which the Modal Accuracy Criterion (MAC) is defined by

$$MAC_i = \frac{\left(\phi_{ai}{}^T \phi_{ei}\right)^2}{\left(\phi_{ai}{}^T \phi_{ai}\right)\left(\phi_{ei}{}^T \phi_{ei}\right)} \tag{5}$$

It has been reported that the modal flexibility is more sensitive to damage than the mode shapes and natural frequencies and then offers a conceptual evaluation [ix,x]. The modal flexibility is the accumulation of the contribution from all available mode shapes and corresponding natural frequencies. The modal flexibility matrix $[F]_{n\times n}$ is defined as [xi]

$$[F]_{n\times n} = [\Phi]_{n\times m} \left[\frac{1}{\omega^2} \right] [\Phi]_{n\times m}^T \tag{6}$$

where $[\Phi]$ is the mode shape matrix and ω is the circular frequency . Similarly, *n* and *m* are number of the measurement DOFs and number of mode shapes considered respectively. If the deflection vector u_i under uniformly distributed unit load, called the uniform load surface, is defined, the objective function Π_3 considering the flexibility residual can be presented as

$$u_i = \sum_{k=1}^{n} \frac{(\Phi_{ik}) \sum_{j=1}^{n} (\Phi_{kj})}{\omega_k^2} \tag{7}$$

$$\Pi_3(x) = \frac{m}{n} * \sum_{j=1}^{n} \left[\frac{u_{aj} - u_{ej}}{u_{ej}} \right]^2 \tag{8}$$

It is necessary to have a mass-normalized mode shapes to use measured flexibility matrix in FE updating which is the major drawback of the procedure. For a force vibration test the mass normalization can be implemented from the driving point inertance_measurement. However, for a modal test that uses an ambient excitation source, the mass-normalized mode shapes are not straightforward. To realize that, the Guyan-reduced mass normalization technique [xii] is used in the paper.

$$\Phi_{ij} = \frac{\varphi_{ij}}{\sqrt{\{\varphi_i\}^T [M] \{\varphi_i\}}} \tag{9a}$$

$$\Phi_{ij} = \frac{\varphi_{ij}}{\sqrt{\sum_{k}^{n} m_k \varphi_{kj}^2}} \tag{9b}$$

The reduction is performed according to Guyan [xiii], which assumes that the inertial forces at the eliminated degree of freedom are negligible. This assumption typically makes this method valid for only the lower frequency modes. The expression shown in equation (9a) is an especially convenient normalization for a general system and for a system having a diagonal mass matrix, it may be written as shown in equation (9b).

Equations (2), (3) and (8) are the objective functions considering frequency residual only, mode shape related function only and modal flexibility residual only. Hence full objective function used in the paper is their combination with the constraints to be imposed on objective functions.

$$\Pi(x) = \Pi_1(x) + \Pi_2(x) + \Pi_3(x) \tag{10}$$

$$0 \le |\lambda_{ai} - \lambda_{ei}| \le UL \tag{11}$$

$$L_1 \le MAC \le 1 \tag{12}$$

where UL is the upper limit whose value can be set as absolute error of jth eigenvalue and

L_1 represents the lower limit to constrain the MAC.

Finite element model updating is carried out to solve a constrained minimization problem whose aim is the minimization of the objective function Π under the constraints. Design variables are subjected to constraints with upper and lower limits that is

$$x = \left[x_1 \ x_2 \ x_3 \cdots\cdots x_N \right] \tag{13a}$$

$$\underline{x}_i \le x_i \le \overline{x}_i \quad (i = 1, 2, 3, \ldots N) \tag{13b}$$

Then the form of optimization is:

$$\text{Minimize} \quad \Pi = \Pi(x) \tag{13c}$$

Subjected to

$$g_i(x) \le \overline{g}_i \qquad (i = 1,2,3,\ldots m_1) \tag{13d}$$

$$\underline{h}_i \le h_i(x) \qquad (i = 1,2,3,\ldots. m_2) \tag{13e}$$

$$\underline{w}_i \le w_i(x) \le \overline{w}_i \qquad (i = 1,2,3\ldots. m_3) \tag{13f}$$

In current optimization algorithm, the penalty function concept is used. Penalty function methods generally use a truncated Taylor series expansion of the modal data in terms of unknown parameters. Three main steps of the of first order optimization method are described as follows:

(a) The constrained problem statement expressed in equation (13) is transformed into an unconstrained one using penalty functions. An unconstrained form is formulated as follows:

$$Q(x,q) = \frac{\Pi}{\Pi_0} + \sum_{i=1}^{n} P_x(x_i) + q\left[\sum_{i=1}^{m1} P_g(g_i) + \sum_{i=1}^{m2} P_h(h_i) + \sum_{i=1}^{m3} P_w(w_i)\right] \tag{14}$$

where $Q(x,q)$ is the dimensionless unconstrained objective function, P_x , P_g , P_h , P_w are the penalties applied to the constrained design and state variables and Π_0 reference objective function value that is selected from the current group of design sets.

(b) Derivatives are formed for the objective function and the state variable penalty functions leading to the search direction in design space. For each optimization iteration (*j*) a search direction vector d^j is devised. The next iteration (*j*+1) is obtained from the following equation (15)

$$x^{(j+1)} = x^{(j)} + S_j d^{(j)} \tag{15}$$

In this equation, measured from $x^{(j)}$, the line search parameter S_j corresponds to the minimum value of Q in the direction $d^{(j)}$.

(c) Various steepest descent and conjugate direction searches are performed during each iteration until convergence is reached. Convergence is assumed when comparing the current iterations design set (*j*) to the previous (*j*-1) set and the best (*b*) set as shown in equation (16)

$$\left|\Pi^{(j)} - \Pi^{(j-1)}\right| \leq \tau \quad \text{and} \quad \left|\Pi^{(j)} - \Pi^{(b)}\right| \leq \tau \tag{16}$$

where τ is the objective function. In this paper, with respect to each parameters, the eigenvalue sensitivity matrix is approximated using the forward difference of function with respect to each parameters considered as shown in equation(17)

$$\frac{\partial f}{\partial x} = \frac{f(x + \Delta x_i e) - f(x)}{\Delta x_i} \tag{17a}$$

$$\Delta x_i = \frac{\Delta D}{100}(\bar{x}_i - x_i) \tag{17b}$$

where, *e*= vector with 1 in its *j*th component and 0 for all other component, ΔD = forward difference (in %) step size, taken 0.2 in this study.

5.2 Updating of the tested bridge

The crucial step is how many parameters to be selected and which parameters from many possible parameters to be considered in updating. If too many parameters are included in the updating, the problem may appear ill-conditioned because only few modes are correctly recognized in the ambient vibration testing. To achieve this objective, the sensitivity analysis is carried out using the maximum number of potential parameters

It is better to start from all possible parameters then identify the most sensitive and non sensitive parameters to response. In this case study of arch bridge, boundary condition is well defined and there is not more uncertainty to boundary condition. Out of possible parameters, eigenvalue sensitivity analysis with respect to initial estimation of parameters is performed for 15 influential parameters. The example of sensitivity analysis is shown in Figure 6. It is found that mass density of arch and deck, material property of deck ,thickness of deck and other selected parameters are most sensitive to most of the modes considered where as the parameters like moment of inertia of arch and area of prestress cable connecting two abutments are not so sensitive. 9 most sensitive parameters are selected for updating purpose.

When selection of parameters and nature and number of mode shapes to be used in updating is confirmed, an objective function and state variables are defined in equation (10~12). The weighting matrix should be chosen in objective function to reflect the relative accuracy among the measured modes. Typically, the frequencies of the lower few modes are measured more accurately than those of the higher modes. If each natural frequency is weighted equally in absolute terms, the algorithm will effectively weight the higher frequency more. By assigning proper values for α_i, the difference between analytical and the measured eigenvalues of the lower modes can be further minimized. In this work, the α_i values corresponding to first four modes are set to be 15 times larger than the remaining modes.

Although it is very hard to estimate the variation bound of the parameter during updating, it can be assumed according to some engineering judgments. The variation of $\pm 20\%$ is given for thickness of deck and $\pm 30\%$ for other parameters

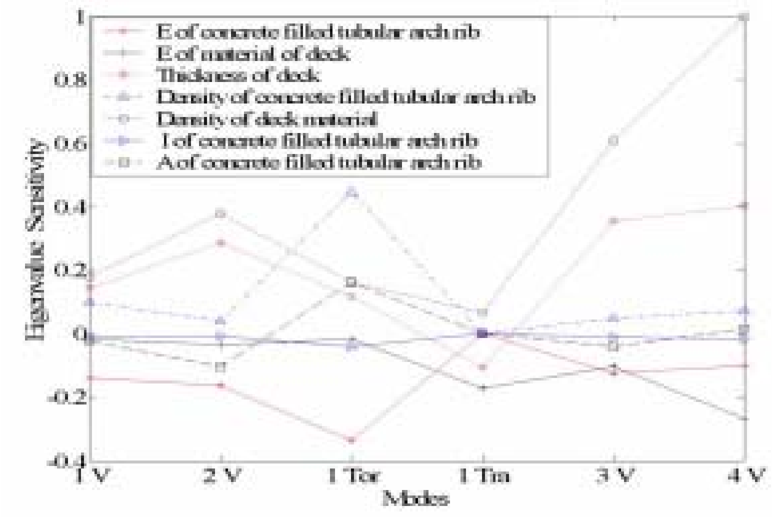

Figure 6: Eigenvalue sensitivity to potential parameters

An iterative procedure for model tuning was then carried out. The selected parameters were estimated during an iterative process. The tuning process is over when tolerances were achieved or predefined number of iterations was reached. Although the optimization process is automatic, many things are required to be considered for successful updating.

The comparison of initial, updated and experimental results summarized in Table 2. It has been shown that the differences between FE frequency and experimental frequency ware reduced below 7% after updating. The errors of the first four frequencies fall below 2%, which is a significant improvement comparing to the initial FE result.

Table 2 : Comparison frequencies (Hz)

Nature of modes	Initial FE	Updated FE	Test results
1st vertical	1.743	1.962	2.002
2nd vertical	2.210	2.493	2.511
1st torsion	2.391	2.815	2.827
1st transverse	2.669	2.737	2.780
3rd vertical	2.778	3.256	3.473
4th vertical	3.541	4.027	3.864

The correlation of mode shape is also improved as all MAC values are over 80% except for first transverse mode which also has improvement on the MAC of 76.6 from initial value

62.1%. Careful inspection of MAC matrix of Figure 7 shows that there is improvement on the MAC value of every mode considered

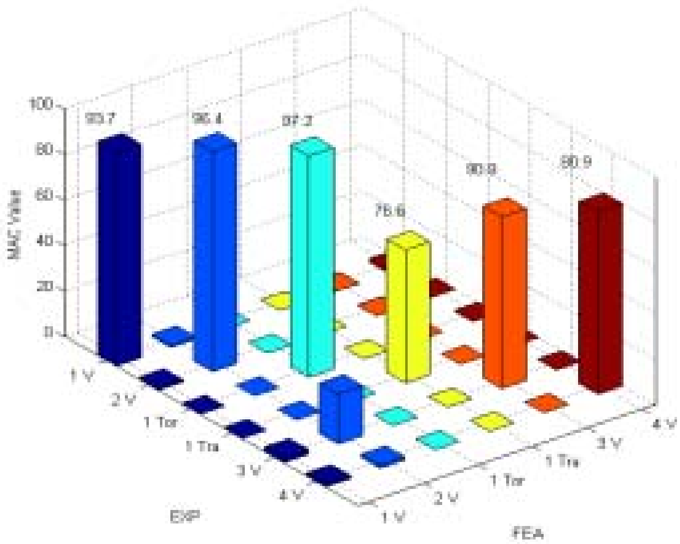

Figure 7: MAC matrix after updating

The change in values of selected parameters with initial and updated values is shown in Table 3. It is demonstrated that the design values are quite different than that of the updated values. The changes in these parameters represented the global change of stiffness and mass matrix

Table 3 : Parameters before and after updating

Parameters updated	Initial values	Updated values	Change
Elastic modulus of arch (Pa)	4.56×10^{10}	5.30×10^{10}	16.3
Elastic modulus of cross girders (Pa)	3.45×10^{10}	4.26×10^{10}	23.5
Elastic modulus of deck (Pa)	3.00×10^{10}	3.90×10^{10}	30.0
Inertia moment of cross girder (m^4)	0.0756	0.0972	-1.76
Thickness of bridge deck (m)	0.25	0.246	3.473
Mass density of arch (kg/m^3)	2871.0	2010.0	-30.0
Mass density of deck (kg/m^3)	2500.0	2144.0	-14.2
Sectional area of arch (m^2)	0.4311	0.3384	-21.5
Sectional area of suspender (m^2)	0.0025	0.0021	-16.0

6 CONCLUSIONS

The paper presented a sensitivity based finite element model updating method for real bridge structures using the test results obtained by ambient vibration technique. Full objective function that considers frequency, mode shape related and modal flexibility residuals. It is demonstrated that the dynamic characteristics of a full scale concrete-filled steel tubular arch bridge can be fully studied by the field ambient vibration tests, the free vibration analysis through three-dimensional finite element method, as well as the finite element model updating by using field ambient vibration measurement results.

REFERENCES

[i] B.C. Chen, *Nonlinear characteristics and ultimate load-carrying capacity of concrete filled tubular arch*, A thesis submitted to Kyushu University, in accordance with the requirements for the degree of Doctor of Engineering, Department of Civil and Structural Engineering, Japan (2003).

[ii] J.E. Mottershead and M.I. Friswell, "Model updating in structural dynamics: A survey", *Journal of sound and vibration*, **167(2)**, 347-375 (1993).

[iii] M.I. Friswell and J.E. Mottershead, *Finite Element Model updating in structural dynamics*, Dordrecht: Kluwer Academic Publishers (1995).

[iv] M. Link, "Updating of analytical models-review of numerical procedures and application aspects", *Proc. of structural dynamics forum SD2000*, Los Alamos (1999).

[v] G. De Roeck and B. Peeters, *MACEC2.0 – Modal Analysis on Civil Engineering Constructions*, Department of Civil Engineering, Catholic University of Leuven, Belgium (1999).

[vi] B. Peeters, *System Identification and Damage Detection in Civil Engineering*. Ph.D. thesis, Department of Civil Engineering, K.U. Leuven, Belgium (2000).

[vii] P. Van Overschee and B. De Moor, *Subspace Identification for Linear Systems: Theory, Implementation and Applications*. Kluwer Academic Publishers, Dordrecht, Netherlands (1996).

[viii] P.W. Moller and O. Friberg, "Updating large finite element models in structural dynamics", *AIAA Journal*, **36(10)**,1861-1868 (1998).

[ix] T. Toksoy and A.E. Aktan, "Bridge condition assessment by modal flexibility", *Experimental Mechanics*, **34(3)**, 271-278 (1994).

[x] J. Zhao and J.T. DeWolf, "Dynamic monitoring of steel girder highway bridge", *Journal of Bridge Engineering*, **7(6)**, 350-356 (2002).

[xi] A. Hoyos and A.E. Aktan, "Regional identification of civil engineered structures based on impact induced transient responses", *Research Rep. 87-1*, Louisiana State University, Baton Rouge, La. (1987).

[xii] S.W. Doebling and C.R. Farrar, "Computation of structural flexibility for bridge health monitoring using ambient modal data", *Proc. of 11th ASCE Engineering Mechanics Conference* (1996).

[xiii] R.J. Guyan, "Reduction of stiffness and mass matrices. *AIAA Journal*, **3**, p.38 (1965).