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THE STONE BRIDGES DESIGN: FROM OLD TREATISES TO NEW NUMERICAL METHODS

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Abstract. *This note is part of a wider departmental research aimed at the analysis of the structural behaviour of the historical constructions.*

The study of treatises has been, and still is, a main point of our research: from Vitruvius to classical treatises (Alberti, Palladio etc.), up to XVII- XIX century treatises. Such an analysis revealed as scientific knowledge has slowly worn away the classical canons by introducing elementary formulae (or tables), in order to obtain the size of the structural elements: this size did no longer depend on the " moduli" harmony but, instead, on the global stability of the buildings and the resistance of materials. A new born "Mechanics of Structures" starts to influence the well established "Ars Aedificatoria".

This paper deals with the rules to design the masonry arch elements and, in particular, takes into account those useful to analyse the late 18th century stone bridges in Campania. Lot of space is given to the "practical rules" stated by the Neapolitan Engineering Vincenzo Lamberti (1740-1790), but we do not neglect, however, to compare them with those found in other contemporary works, which were widespread in the Neapolitan cultural environments.

With reference to another paper dealing with typology and building methods of Roman bridges and bridges from the 17th to the first half of 19th century in southern Italy, we analyse, here, some arch bridge in detail, in order to verify not only to what extent new rules are set into building tradition inherited by Romans, but also their validity in the light of the present knowledge. To this purpose, thanks to a simple numerical method , the limit load and the "collapse mechanism" of the chosen examples are calculated, modelling the masonry arch by means of no-tension rigid blocks and neglecting mortar contribution.

1 INTRODUCTION

The work on "The masonry Bridges in the South of Italy: From the Roman Tradition to the First half of the XIX Century", by L. Bove, I. Bergamasco and M. Lippiello, illustrates all bridge typologies and construction techniques in relation to their cultural, political, social and economical contexts. The discussion also highlights how the alternation of development and innovation periods and those of carelessness and indifference depended on the different policies that were brought forth in the South of Italy.

The historical account comes to a halt at the beginning of the XIX century. Such an interruption is intentional because it allows for a detailed analysis of a complex moment: the progressive substitution of well established construction practices with the prescriptions of the rising mechanics of materials and structures.

The Campania region is the main focus of this discussion. In the period going from the end of the XVIII to the beginning of the XIX century, this area was strongly influenced by French culture and the Neapolitan scientists, given their strong pedagogical and didactic commitment, played an important role in the spreading of the new doctrines and in the development of their practical application. The military schools, more than the Universities, aimed at a profession-oriented education, a specialized training, and therefore became the main source of original knowledge within the applied sciences. In De Sanctis' words: :«[...]le accademie militari che istituzionalmente erano legate al re, erano paradossalmente molto aperte alle nuove correnti culturali e spesso svolgevano un ruolo avanzato anche dal punto di vista politico. Teorie e metodi tecnico-scientifici aggiornati vi circolavano liberamente»"[1].

The above mentioned work on "The masonry Bridges in the South of Italy" described the cultural wealth of the Neapolitan XVIII century and analysed its complex scientific environment; the discussion that follows will focus on a particular issue within this framework: the relationship between a new knowledge brought forth by the teaching efforts of important scholars and the practical applications.

2 THE TRAINING OF EXPERTS

The "ancient books" section of the Engineering Faculty of the University of Naples Federico II bares proof of the rich Military School cultural heritage dating the XVIII and XIX centuries. The collection of texts shows how students were trained on the basis of updated Italian and foreign scientific tracts and further knowledge came from the Engineer Corps and the school teachers. New works such as: *Traité de la coupe des pierres* by Jean-Baptiste de la Rue, *Traité analytique de la résistance des solides et des solides d'égale résistance* by Pierre-Simon Girard, *Science des ingénieurs dans la conduite des travaux de fortification et d'architecture civile* and l'*Architecture Hydraulique, ou l'art de conduire, d'élever et de ménager les eaux pour les différent besoins de la vie* by Bernard Forest de Bélidor, and volumes belonging to the Vitruvian tradition stood side by side and, therefore, reflected those existing contradictions and dichotomies that the cultural debate of the time transferred to the domain of professional practical application.

Nonetheless, it is clear that notions deriving from scientific disciplines were added to the standardization of the elements handed down by the classical tracts: so that the engineers and architects could know in advance how the structure would behave before its actual construction. It was an attempt to reach that condition of "knowing before doing" auspicated by Galileo in his *Discourses*.

3 THE TRACTS ON BRIDGES

There were a certain number of studies, circulating in the Campania region, which defended the need to give scientific support to the formalization of construction modes. Reference to these tracts is a necessary step for the analysis of the shift from the "rules of art" to the "new science" and how this shift influenced the design of masonry bridges.

A special attention as to be paid to the "Traité des pontes", the first specific text on the subject, written by H. Gautier[2], an architect, an engineer and, last but not least, an inspector Des Ponts & Chaussées du Royaume. At the end of the essay's first chapter, after a re-discussion of what the classical tracts said on the proportions of single parts, the author complains about the lack of ascertained rules for construction planning. In his own words: "C'est la tout ce que les plus habiles Architectes nous ont donné par écrit de la proportion des Ponts, mais pour nous donner des raisons démonstratives, personne ne l'a pas fait encore; [….] Autant d'Architectes, autant d'avis différent; ils ne nous donnent aucune raison pourquoi ils font les Piles, les Culées, les Arches, etc., d'une telle largeur, ou d'une telle épaisseur, & ceux qui travaillent aujourd'hui sur les exemples des Anciens, ne savent pas non plus pour quelle raison ces Auteurs ont travaillé ainsi». The author adds that de la Hire did try to solve some these issues but because of his abstract mathematical language he could not be understood by the operators and, therefore, his work was useless in this respect.

In the XXX chapter Gautier write down the: "Cinq difficultés qu'on propose aux Savants a résoudre" with the request that scholars solve and demonstrate them. But, shortly after, we find the warning that: « Les hypothèses qu'on établira pour principes, doivent être connues, certaines, évidentes, & dont on ne puisse pas douter. On demande qu'on s'explique avec des termes & un langage connu, afin que tout le monde l'entende, & en puisse juger».

Only in the second edition of the treat he goes deeply into the problems, adding to the original text the "Dissertation sur les culées, voussoirs, piles et poussées des ponts".

In the first chapter of his dissertation the author lists again the five problems to which he tries to find a solution in the eight following chapters:

1. the thickness of the abutments piers in relation to the arches' span and to the weight that they sustain;
2. the dimension of the internal piers in relation to the arches' span and to the weight above them;
3. the thickness of the voussoirs between intrados and extrados in the neighbourhood of the keystone in relation to the size of the arches;
4. how to determine, in relation to the same span, a suitable shape of arch, able to sustain maximum weight;
5. how to determine the best profile for the retaining walls.

A list of the simple means necessary for the understanding of these five points follows: the basics of physics for the understanding of the relationship among different materials, the basic static laws in order to guarantee the balance of the parts, the basics of mechanics in order to measure the forces directed to the fixed elements which are supposed do not deform, and, last but not least, notions of geometry in order to be able to measure the surfaces and volumes of the bodies. The author's conclusion is the following: « Un peu de chacune de ces Sciences avec le sens commune, suffiront pour faire connaître facilement ce que j'avance. Je n'oublie rien pour me rendre aisé & intelligible, afin que ceux qui ne savent pas, pour qui uniquement je travaille, en puissent plus facilement juger ».

Given the brevity of this discussion, we will not give a detailed presentation of the dissertation's single chapters. Instead we will highlight some significant characteristics of the work in general.

Each topic is anticipated by a re-analysis of the classics and by a critical synthesis of the current scientific views of the time. On the basis of this framework, Gautier develops his demonstrations in a discursive manner by reducing to a minimum the algebraic and geometric apparatus necessary for their understanding and by comparing, when possible, these results with those of the "rules of art".

The shape of the arcades, the piers' and abutments' measures cease to be determined exclusively by geometrical relations and abstract reasoning on ideal forms. Instead, such issues are solved also thanks to evaluations of water floods, of river beds and of the single elements' thrust and resistance.

Balance problems are still solved with the support of elementary means – lever, wedge, inclined plane – and the constructing material is considered undeformable. However, the consciousness of a limited resistance requires detailed experimental campaigns in order to determine the characteristic values of each kind of stone. As matter of fact, Gautier adds that: «[…]nous n'avons encore trouvé aucune règle de les mécaniques qui puisse le déterminer, faute d'expériences», but he also warns that the experimental data have to reduced by ¼ in order to compensate the handcraft defects and the presence of joints.

Furthermore, the results of such approach are synthesized in tables in which he assigns the right width to abutments, piers and voussoirs in relation to arch spans and stone resistance, even if the established values, sometimes, take into account the classic data more than the theoretical formulas.

The need to formalize planning rules within the framework of a scientific theory which could also be easily handed down to those architects and engineers who were foreign to this type of research, the opportunity of determining the minimum width of the sustaining elements in order to avoid useless and expensive increases of proportions and, last but not least, the need to know and record the differential stone resistance are the main goals of many following studies which were published up until the second half of the XIX century. A significant example of this type of research was an essay by Gauthey[3], posthumously edited by Navier.

4 VINCENZO LAMBERTI

Vincenzo Lamberti, Neapolitan engineer, can be considered a peculiar figure in the theoretical-practical architectonic survey of Campania region in 18th century: the local treatisers (as Carletti) and the main architects working on the area (as Vanvitelli), even if aware of the scientific advances, remained faithful to classical models and refused the explicit contribution of the new sciences.

Lamberti, on the contrary, moulded in the Military School, joined the new static's theories and, following French examples, tried hard to translate these into "practical rules", useful to the professional architects, less conscious of mechanic's and geometry's new trends.

This aim gave rise to the tracts - "Voltimetria retta"[4] and "Statica degli edifici"[5] – which should be followed by the texts, never published, on "Voltimetria scalena", on the static of slanted vaults, on the stone bridges and other typologies frequently occurring in constructions.

Particularly interesting, for our purposes is the "Statica degli edifici". In the introduction he criticized Architects for giving too much weight to Vitruvius's *venustas and concinnitas* sacrificing the *firmitas* in constructing buildings either too weak or uselessly over-dimensioned. He contested the theoretical foundations of treatises of the past: solidness and stability don't derive naturally from the respect for eurhythmy , for the module or symmetry nor are "imperfections of the materials" the cause of fractures. Bearing in mind Galilei's assumption that matter does not obey "abstract and ideal reasoning", Lamberti was convinced that only experimental data of the resistance of materials and an accurate use of mathematics ensured the stability of buildings.

On the basis of known experiments (Mariotte, Parent, Musschenbroek, etc.), he assumed a priori that "there was no constant ratio between the *absolute force* - breaking load under traction – and the *relative force* - breaking load under bending stress" and organised a series of experiments to determine the values of the "relative resistance" of the various materials used in Campania: tufa, piperno, lime and pozzolana. The tests were carried out on small prismatic samples with square base, fixed at one end and loaded at the free base, or resting on both ends and loaded at midpoint. The experiment data were then revised (using the equilibrium of the angular lever and the hypothesis of constant distribution of the resistance on the section of maximum stress), in order to obtain values independent of sample geometry and of test methods. Lamberti thus obtained the following values of the breaking load of cube samples with the side of 1 Neapolitan palm (26,3cm), fixed at one end and loaded at the free end: Campania tufa – *rotoli* 1873 (1668 Kg); piperno – *rotoli* 10080 (8981 Kg); lime and pozzolana – *rotoli* 939 (837 Kg).

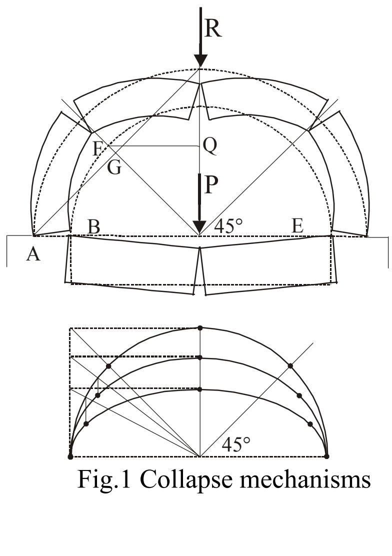

Fig.1 Collapse mechanisms

These elements were sufficient for him to obtain theoretically, with only the aid of geometrical similarities, the necessary formulae to dimension the beam elements under different constraint and load conditions. Particularly original and interesting are the rules concerning the determination of the ultimate load, R, of arches, semicircular and not, obtained by analogy with that, P, of a beam of the same material and thickness and with a length equal to the span of the arch itself. The formula he obtained for the semicircular arch,

$$\frac{P}{R}=\frac{3}{5}\frac{FQ+AG}{BE} \tag{1}$$

is vitiated by the theory, at the time still predominant, that the breakage of the reins corresponds to the slanted voussoir at 45° and by some questionable algebraic transformations; besides, there seems to be little ground for the ratio 3:5 that multiplies the geometric data which, for Lamberti, represents the effect of the different number of fractured sections that characterize the collapse mechanisms of the two structural elements compared.

The resistance of the segmental arch is obtained by a similar formula and the weakest joints are determined as shown in fig.1: it can be noticed how the collapse sections tend towards the springer sections until they coincide with them when the arch joins the beam.

In the analysis of arch and vault thrusts, Lamberti singles two types of problems:

a. the minimum thickness to give to the vault so that it resists to its own weight and of the elements loaded on it;
b. the thickness of the piers.

In the formulation of the first type of problems, he essentially applies formula (1), or its derivatives for the segmental arch, obtaining firstly the load bearing on the arch and then the thickness of the beam of equal length to the free span.

The way he delves into the type b problems seems more interesting. The theoretical instrument is again that of the angular lever and the end formulae, worked out by long chains of geometric similarities, are in fact revisions and adaptations of the ones obtained in the analysis of isolated walls.

The originality of his approach, in our opinion, is to be found in his conclusions: for each significant type of vault, the author numerically solves a standard problem; from this, he obtains factors that don't change with geometric data variations of the single problem and hence defines a new *practical* calculation procedure combining these constant terms with the variable data.

For example, to determine the thickness of the abutment of a round barrel vault, after calculating the dead force P exerted by the arch, Lamberti arrives at a first value "a" of the thickness by applying the formula previously obtained for the isolated wall. In the case examined, however, force P does not pass through the pier edge and hence the solution must be corrected to determine the new thickness, x+a, to re-establish balance. Thus, the following expression is obtained

$$x=\frac{a\times m\times n}{a\times h+a\times m} \tag{2}$$

that measures the required thickness.

Applying the above procedure to a vault made of Campania tufa with a radius of $r_m = 8$ palms and an impost height of $h_m = 24$ palms, Lamberti obtains the dead force $P_m = 23{,}17$ *rotoli* and the thickness of the pier $a_m = 6{,}4$ palms.

These data are sufficient to determine the thickness of the abutment of any round barrel vault made of Campania tufa with the formula *-pratica I-*

$$y = \sqrt{\frac{P \times r \times h_m \times a_m}{P_m \times r_m \times h}} = \sqrt{\frac{P \times r \times 24 \times 6{,}4}{23{,}17 \times 8 \times h}} \tag{3}$$

where r, h and P stand for the radius, the impost height and the dead force.

Similarly, he arrives at six other *praticas*, founded on six basic numerical solutions covering the different types of barrel vaults, with or without piers exceeding the imposts, and flat arches.

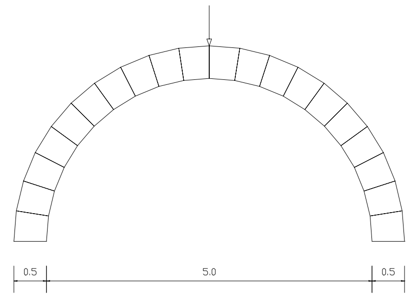

Fig. 2 Arch geometry

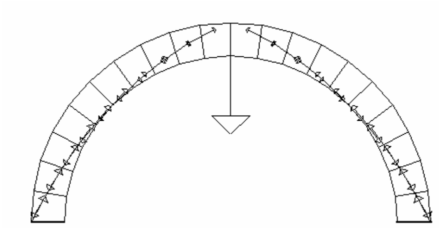
Fig.3 Thrust line

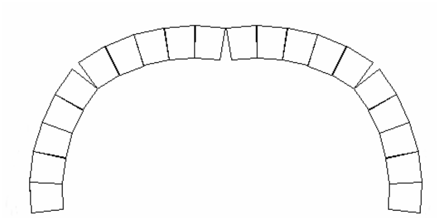
Fig.4 Collapse mechanism

The central role played by (1) in the whole treatment induces to verify its validity. The perplexities on its derivation were already underlined: the formulation of the problem doesn't result clear, some algebraic transformations appear doubtful, the corrective factor 3/5, theoretically, is not justified. To test the reliability of the arch's critical load, given by Lamberti's procedure, the problem is solved also using the usual 'engineering' assumption[6] made for masonry: no tensile strength, virtually infinite compressive strength, slip does not occur between components of the structure. The two described approaches, applied on circular arches with different ratio thickness/span, give the results plotted in table 1,

t	**s**	**Lamberti's P_c**	**Matwall's P_c**
0.5 m	4 m	4708 Kg	5562 Kg
0.5 m	5 m	3148 Kg	3362 Kg
0.5 m	6 m	1933 Kg	2202 Kg

Table 1

where **t**, **s** and $\mathbf{P_c}$ denote the thickness, the span and the limit load of the arch respectively; and Matwall is the used numerical procedure that will be illustrated in the next section. Applying Matwall to the arch of fig. 2 , the thrust line and the collapse mechanism of figures 3 and 4 are obtained.

Results show that the ratio Pc Lamberti/ Pc Matwall is 9% approximately. It is than possible to suppose that the above underlined theoretical inconsistencies are used from the Author to match theory with practise: the same attitude was previously pointed out commenting on Gautier's tables.

5 NUMERICAL METHOD

The generic element c_m is a convex region bounded by the n sides i-j of the $p_{m,n}$ polyline, oriented clockwise (fig. 5); for each element we introduce the following coordinate systems: the global system $O(x_1, x_2)$ and the local systems $O_i(x_{n,1}, x_{n,2})$ relative to side i-j (fig. 6). Indicating with:

$G_m \equiv (x_{mG,1}, x_{mG,2})$	centre of mass of element c_m;
$g_{m,n} \equiv (x_{mng,1}, x_{mng,2})$	mid point of side n of element c_m;
$L_{m,n}$	length of generic edge of polyline $p_{m,n}$;
$d_{m,n}$	distance from the mass centre and mid side point of the generic edge.

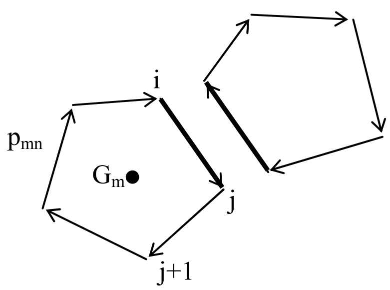

Fig. 5 Oriented polyline

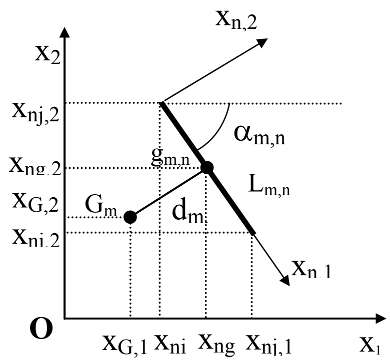

Fig. 6 Reference systems

The generic element can have: restrained, loaded, free and interface edges. Reaction forces and contact forces, $\mathbf{Q}_{m,n} \equiv [\mathbf{x'}_{mn1}, \mathbf{x'}_{mn2}, \mathbf{x'}_{mn3}]$ applied in the mid point g of restrained and interface edges are defined positive in local coordinate system. The moment is positive if counter clockwise (fig. 7).

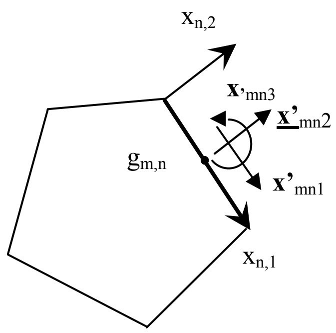

Fig. 7 Contact forces

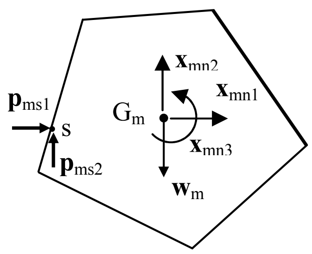

Fig. 8 Forces expressed in global system

Weight $\mathbf{w}_m$ is applied in the centre of each element and is estimated automatically by the nodal coordinates, the thickness and the density γ.

Expressing all local forces in the global system thought the rotational operator, we have (fig. 8):

$\mathbf{Q}_{m,n} \equiv [\mathbf{x}_{mn1}, \mathbf{x}_{mn2}, \mathbf{x}_{mn3}]$	Reaction and contact forces $\mathbf{Q}_{m,n}$ expressed in the global system and evaluated at G_m;
$s \equiv (x_{ms,1}, x_{ms,2})$	Applied force position in global coordinates;
$\mathbf{p}_{ms} \equiv (\mathbf{p}_{ms1}, \mathbf{p}_{ms2})$	Load components;

Looking for the critical load and denoting by "h" the number of edges for elements, "r" the number of applied loads $\mathbf{p}_{ms}$ on each element and "k" the number of elements, it is possible to define the following LP problem:

Equilibrium equations

$$\sum_{m=1}^{k}\sum_{n=1}^{h} x_{mn1} + \lambda \sum_{m=1}^{k}\sum_{s=1}^{r} p_{ms1} = 0$$

$$\sum_{m=1}^{k}\sum_{n=1}^{h} x_{mn2} + \sum_{m=1}^{k} w_m + \lambda \sum_{m=1}^{k}\sum_{s=1}^{r} p_{ms2} = 0$$

$$\sum_{m=1}^{k}\sum_{n=1}^{h}\left[x_{mn2}\left(\mathrm{x}_{mng.1} - \mathrm{x}_{mG,1}\right) + x_{mn1}\left(\mathrm{x}_{mng.2} - \mathrm{x}_{mG,2}\right) + x_{mn3}\right] +$$
$$+ \lambda \sum_{m=1}^{k}\sum_{s=1}^{r}\left[p_{ms2}\left(\mathrm{x}_{ms.1} - \mathrm{x}_{mG,1}\right) + p_{ms1}\left(\mathrm{x}_{ms.2} - \mathrm{x}_{mG,2}\right)\right] = 0$$

Limit conditions

$$\frac{\mathbf{x}'_{mn3}}{\mathbf{x}'_{mn2}} \leq \frac{L_{mn}}{2}$$

$$\frac{\mathbf{x}'_{mn3}}{\mathbf{x}'_{mn2}} \geq -\frac{L_{mn}}{2}$$

Normal stress vector sign limit

$$\mathbf{x}'_{mn2} \geq 0$$

Objective function

$$\lambda = \max$$

that can be solved automatically with the LP programming algorithms of MatLab.

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