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DEVELOPMENTS TO THE RING MASONRY ARCH BRIDGE ANALYSIS SOFTWARE

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Abstract. *The RING computational limit analysis software for masonry arch bridges was originally developed as a research tool in the early 1990's but has been publicly available via an internet site since 2001. This paper describes recent and planned future developments to the software which is currently being completely rewritten, including: order of magnitude improvements in its computational efficiency; a non-associative constitutive model for sliding friction; enhancements to the general usability of the software. The new software is being developed using an object-oriented approach so that further significant developments (e.g. 3D arch analysis and full coupled arch-soil limit analysis) should be able to be readily incorporated in the future.*

1 INTRODUCTION

Heyman[i] pointed out that despite the lack of plastic moment capacity of the sort steel and reinforced concrete structures possess, plastic limit analysis methods of analysis could justifiably be applied to masonry gravity structures, such as piers and arch bridges. The complexity of arch bridge geometry coupled with the complex loading patterns typically found in practice means that hand-based limit analysis is not generally practicable. Additionally, some of the assumptions made originally by Heyman[i] are no longer considered generally acceptable (e.g. infinite masonry crushing strength; no sliding failures). Hence various computer-based mechanism methods of analysis have been proposed, these methods using either rigorous or ad-hoc optimisation techniques to identify the critical failure mode. Amongst these, the rigid block computational limit analysis method[ii,iii] is the most generally applicable and may for example easily be applied to arch problems involving multiple arch rings and/or spans.

Currently the only widely available software based on the rigid block computational limit analysis method is RING, software which was developed originally as a research tool[iv]. Since the first publicly available version of RING was released in January 2001 (version 1.1), the software has proved remarkably popular worldwide, particularly with practitioners (the software can be freely downloaded from the web at **`http://www.shef.ac.uk/ring`**).

However, parts of RING version 1.1 date back more than a decade and it has now become difficult to enhance and extend the existing software without expending effort disproportionate to the benefit gained. Hence the decision has now been taken to rewrite the software from the ground up. This has also stimulated a review of all parts of the existing software. Thus there are three main issues which have been identified as priorities for the forthcoming releases of RING: (i) improved speed of execution (particularly for multi-ring arch problems); (ii) enhanced realism of the computational model (e.g. better modelling of friction, soil-structure interaction etc); (iii) other enhancements to improve the usability of the software.

This paper considers each of the these issues in turn. To facilitate the efficient re-programming of the software, the popular object-oriented programming language C++ is being used (it is intended that the program source code of the new software will also be made available for interested parties to inspect/modify as required).

2 EXECUTION SPEED

RING 1.1 (and earlier research versions of the software) used a redundant forces limit analysis problem formulation (or, strictly speaking, the ‘dual’ of a redundant forces formulation, expressed in terms of problem kinematics). This led to a very compact but potentially very densely populated linear programming (LP) constraint matrix. Although most simple single and multi-span problems could be solved almost instantaneously as far as the user was concerned on a modern PC, it was found that execution times for more demanding multi-ring problems were often undesirably long, even when a state-of-the-art PC was employed. Part of the problem stemmed from the fact that the original formulation was tailored for the Simplex LP algorithms available in the early 1990’s yet RING 1.1 was necessarily distributed with a freely available interior point LP solver more suited to solving large, sparse, problems.

In recent investigations it has been found that although a conventional joint equilibrium formulation[ii] produces a large number of constraints and variables, the total number of non-zero elements is generally relatively small, which means that it can be solved very efficiently using modern interior point based LP algorithms. This problem formulation may be concisely stated as follows:

$$\text{Max } \lambda \tag{1}$$

subject to

$$\mathbf{Bq} - \lambda \mathbf{f}_L = \mathbf{f}_D$$

$$\left.\begin{aligned} m_i &\le 0.5 n_i t_i \\ m_i &\ge -0.5 n_i t_i \\ s_i &\le \mu_i n_i \\ s_i &\ge -\mu_i n_i \end{aligned}\right\} \quad \text{for each contact, } i = 1,\dots,c$$

where there are b blocks and c contact surfaces in the problem, λ is the load factor, $\mathbf{B}$ is a suitable ($3b \times 3c$) equilibrium matrix derived from the geometry of the structure and $\mathbf{q}$ and $\mathbf{f}$ are respectively vectors of contact forces and block loads. Thus $\mathbf{q}^{\mathbf{T}} = \{n_1, s_1, m_1, n_2, s_2, m_2, \dots n_c, s_c, m_c\}$; $\mathbf{f} = \mathbf{f}_D + \lambda \mathbf{f}_L$ where $\mathbf{f}_D$ and $\mathbf{f}_L$ are respectively vectors of dead and live loads. Contact and block forces, dimensions and frictional properties are shown on Figure 1. Using this formulation the linear programming problem variables are clearly the contact forces: n_i, s_i, m_i (where $n_i \ge 0$; s_i, m_i are free variables).

It should be noted that implicit in the above formulation is an associative, or 'saw-tooth' type friction model (i.e. stipulating that dilatancy accompanies sliding). This issue will be returned to in section 3. Additionally in order for masonry crushing to be accommodated an iterative analysis in which the effective contact length is modified at successive iterations is required (e.g. as discussed in a previous Arch Bridges conference paper[v]).

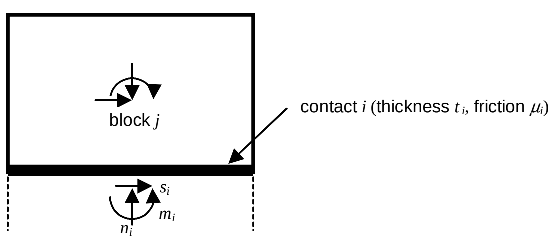


Figure 1: Block and contact forces

2.1 Multi-ring arch analysis example problems

In order to demonstrate the comparative computational efficiency of the above formulation, the 3m and 5m span single-span arch ribs and bridges tested by Melbourne and

Gilbert[vi] will be modelled numerically. The bridges of interest here contained detached spandrel walls and the defect of ring separation, this being achieved in the laboratory by using dampened sand in place of mortar between the rings.

Table 1 shows sample RING analysis results for these bridges. To obtain the RING results a standard coefficient of lateral earth pressure was specified (rather than the back-substituted experimentally recorded pressures, as used in the original publications[vi]). A lateral earth pressure coefficient value of 4.5 was used (approximately equal to $K_p/3$ when K_p is calculated using classical vertical wall passive pressure theory with the measured backfill internal angle of friction of 60°). Additionally a Boussinesq type model was used to simulate dispersion of the applied load through the fill.

Also shown on Table 1 are run times when using a 1.4GHz Pentium 'M' (Centrino) PC. In all cases the PCx[vii] interior point linear programming solver distributed with RING was used, rather than a commercial solver as used in the original publications (in general commercial solvers are more efficient, particularly in the case of redundant forces formulation problems).

Bolton arch rib / bridge ref.	Expt. Failure load (kN)	RING 1.1 (redundant forces formulation)		RING 1.5 (joint equilibrium formulation)		Difference: RING 1.5 vs. 1.1	
		Failure load (kN)	CPU time (s)	Failure load (kN)	CPU time (s)	Failure load increase	CPU speedup factor
Arch 2	1.5	1.44	6.1	1.45	0.16	0.7%	38×
Bridge 3-2	360	252	7.9	253	0.25	0.3%	31×
Bridge 5-2	500	482	236	486	0.98	0.8%	241×

Table 1: Multi-ring arches: experimental and infinite crushing strength analysis results with redundant forces and joint equilibrium formulations

From Table 1 it is evident that when using PCx the speedups associated with the use of a joint equilibrium formulation are very significant, more than 200× in the case of Bridge 5-2. It is also notable that the computed failure loads are slightly greater when using the joint equilibrium formulation. This is because whereas previously a joint between adjacent rings was idealized using a series of point contacts, with the joint equilibrium formulation all contacts in the problem are treated identically, whether these lie in radial or circumferential joints (i.e. all treated as surfaces). In all cases the computed capacities are lower than the experimentally recorded values. This is likely to at least partly result from the simplified soil model employed. Additionally it should be noted that, for reasons which remain unclear, in the case of bridge 3-2 the experimentally recorded collapse load appears very high given that a nominally identical bridge tested with attached, rather than detached, spandrel walls failed at a much lower load (320kN).

2.2 Incorporation of joint equilibrium formulation into RING 1.5

The significant improvements in computational efficiency attributable to the use of a joint equilibrium formulation when used in conjunction with an interior point LP solver has meant that this has now been implemented in a new RING release, version 1.5. This is essentially an interim release pending the release of the completely rewritten version of the software (RING 2.0, to be released in early 2005). For problems involving finite masonry crushing strengths, the reduced solution times have also meant that in RING 1.5 finite masonry crushing strength calculations can now be performed for all load cases in multiple load case problems in a reasonable time. Previously a crushing analysis was just performed for the load case found to be critical following initial, infinite crushing strength, analyses for all load cases; this sometimes led to a non-conservative load factor being computed, e.g. refer to Figure 2 for an example recently encountered in practice.

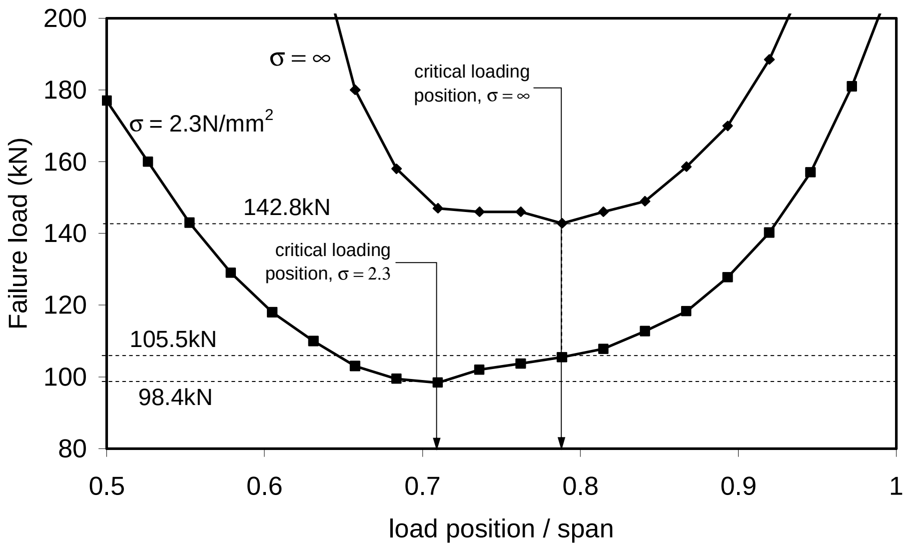


Figure 2: Computed axle load at collapse vs. position

Examining Figure 2 reveals that here the critical load positions for the $\sigma = \infty$ and $\sigma = 2.3\text{N/mm}^2$ cases do not coincide. Taking the critical load position for the $\sigma = \infty$ case and performing crushing calculations only for this load case leads to a predicted capacity of 105.5kN, overestimating the actual carrying capacity of 98.4kN by 8.5 percent.

3. REALISM OF COMPUTATIONAL MODEL

In terms of complexity, computational limit analysis (CLA) models are positioned midway between hand-based methods and (non-linear) elastic analysis tools. Advantages of CLA are that relatively modest operator expertise is required, and that certain classes of problems regarded as ‘difficult’ (e.g. uniaxial contact problems) can be solved very simply and quickly using LP solvers which are widely available and improving in efficiency year-on-year.

However, there are problems. For example, when finite material strength is involved the problem formulation becomes *non-linear* and hence, if LP solvers are still to be used, an iterative analysis procedure is required. Additionally, whilst RING versions 1.1 & 1.5 are capable of modelling sliding failures, it is assumed that associative (or 'saw tooth') friction exists between the constituent blocks. This is not entirely realistic and an arguably more realistic, but again unfortunately *non-linear*, formulation is discussed in §3.1. In §3.2 the very important but rather neglected issue of soil-structure interaction is also briefly considered.

3.1 Modelling sliding failures

Drucker[viii] pointed out that assuming associative (or 'saw tooth') friction between smooth blocks will, in general, lead to upper bound (unsafe) load factors. Despite this, it was previously found by the first author that when modelling multi-ring brickwork arch bridges reasonably good agreement between experimental and numerical results could be obtained when associative friction was assumed (in fact it was found that the numerical multi-ring model always under-estimated the experimentally observed carrying capacity).

Since then, a number of workers have proposed algorithms to model non-associative friction (e.g. refer to reference [ix] for details). Unfortunately the non-associative problem becomes essentially a combinatorial one, with considerable computational expense required to identify minimum load factors for real-world problems. Furthermore, except in trivial cases, these load factors cannot be guaranteed to represent true lower bounds on the actual load factor yet, at the same time, may in fact grossly underestimate the capacity of a real structure which, to use a suitable anthropomorphism, is not necessarily 'clever enough' to identify the worst case load path identified numerically.

In spite of these difficulties, the present authors have recently developed a conceptually simple and comparatively computationally inexpensive procedure for treating non-associative friction problems[x], with a view to offering this as an option in RING 2.0. What follows is a brief description of the method and its application to multi-ring arch problems.

(a) A simple procedure for non-associative friction

In the proposed procedure, rather than make use of highly complex and specialized mathematical programming algorithms as others have done, only a standard linear programming solver is required. Central to the thinking behind the method is the fact that, when using linear programming to solve limit analysis problems, flow will always occur normal to the specified failure surface (i.e. according to the so-called 'normality rule'). The proposed procedure starts with an initial associative friction analysis. Then, to avoid unwanted dilatancy, a subsequent analysis is performed using a new failure surface, formed by rotating the original failure surface about point (n, μn) until it is orientated horizontally (where n is the normal force from the previous iteration; refer to Figure 3). The procedure continues, using successively modified failure surfaces, until a converged solution is obtained. Details of a number of minor modifications to this basic procedure in order to improve convergence are provided elsewhere[x].

(b) Non-associative friction examples

The benchmark in-plane block wall problems used by Ferris and Tin-Loi[ix] were initially investigated using the procedure[x]. For these problems it was found that when using a non-associative (and zero dilatancy) friction model, predicted load factors were up to approx. 25

percent lower than their associative friction counterparts. Whilst the load factors obtained using the proposed method were never as low as the published MPEC results[ix], they were always within 2 or 3 percent of these.

The proposed method is here applied to multi-ring arch problems for the first time. Thus the arches considered in §2.1 are now re-analysed assuming non-associative friction (and zero dilatancy). Table 2 presents the main results whilst obvious visible differences in the failure modes are highlighted in the case of bridge 5-2 on Figure 4.

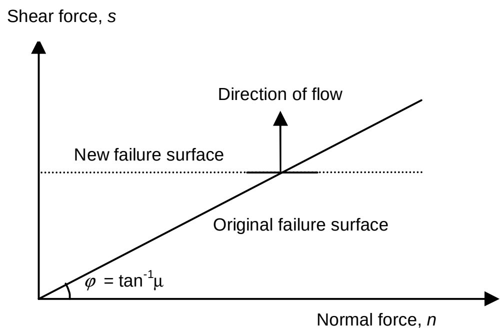


Figure 3: Non-associative (zero dilatancy) sliding friction: original and modified failure surfaces

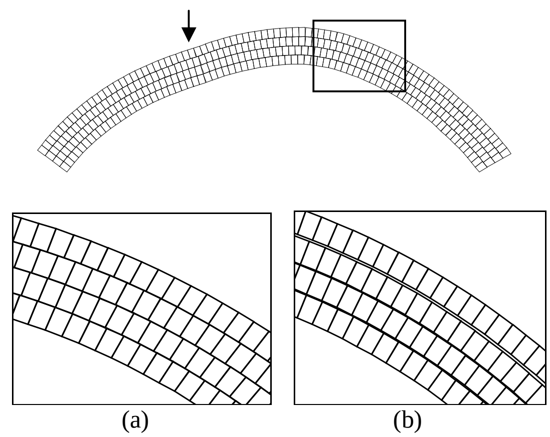


Figure 4: Bridge 5-2 predicted failure modes: (a) non-associative friction; (b) associative friction

Bolton arch rib / bridge ref.	Expt. Failure load (kN)	RING 1.5 (associative friction formulation)		RING: output from non-associative friction procedure		Difference: non-associative vs. associative	
		Failure load (kN)	CPU time (s)	Failure load (kN)	CPU time (s)	Reduction in failure load	CPU slowdown factor
Arch 2	1.5	1.45	0.16	1.44	1.6	0.7%	10×
Bridge 3-2	360	253	0.25	248	1.3	2%	5.2×
Bridge 5-2	500	486	0.98	457	24.1	6%	25×

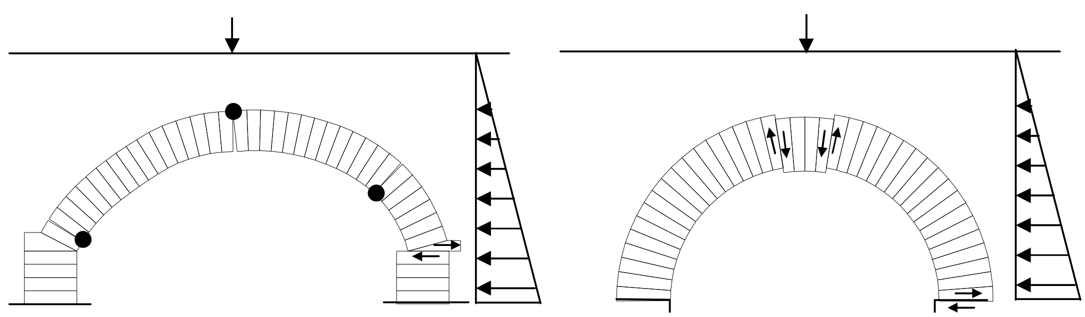
Table 2: Multi-ring arches: experimental and infinite crushing strength analysis results with associative and non-associative friction models

For the cases detailed in Table 2 it is evident that modest reductions in the computed collapse load result from the use of the non-associative friction model (up to 6%). It is also evident that considerable extra computational effort is required in order to obtain the results (up to 25× more CPU time required).

3.2 Soil-structure interaction

It is now reasonably well established that the ultimate load carrying capacity of a soil-filled masonry arch bridge is often highly dependent upon the properties of the backfill. This is because the backfill acts both to disperse the applied loading and to restrain sway of the barrel into the fill. Conventional limit analysis (and many other) models often suffer from the fact that unless the backfill is modelled explicitly, backfill pressures restraining the masonry generally need to be stipulated in advance of an analysis (yet in reality these will be a function of the failure mode, which is not known in advance).

To illustrate this it is worthwhile to consider some sample problems using RING. This software, in common with other masonry arch bridge analysis programs, has been calibrated against results from full-scale bridges, which, for various reasons have tended to fail in 4 hinge mechanisms. In RING the presence of uniaxial backfill elements[iv] means that although it is unnecessary to specify in advance the sense of the pressures, the *magnitudes* of the pressures do need to be specified in advance. Thus in order to approximately reproduce the results from full-scale tests, horizontal passive zone restraining pressures might commonly be entered as $K_p/3$ where $K_p = (1 + \sin\phi')/(1 - \sin\phi')$, and where ϕ' is the internal angle of friction of the backfill material. However, RING chooses the critical failure mechanism from a multitude of possible ones and a 4 hinge failure mechanism is by no means always identified as being critical. For example, Figure 5 shows two failure modes encountered when recently assessing a number of local authority owned field bridges.



(a) (b)

Figure 5: Non-standard predicted failure modes: (a) 3 hinges & abutment sliding; (b) sliding only mechanism

Both the predicted failure modes shown in the Figure involve sliding failures (non-associative failure modes are shown but in these cases the predicted non-associative failure loads were identical to their associative friction counterparts). In the case of the bridge shown on Figure 5(a), despite the fact that horizontal restraining pressures were in this case applied to the back of the skewback, the latter was predicted to slide. However, the magnitudes of the pressures specified were calculated using modified classical vertical retaining wall theory as outlined above whereas in reality such a failure mode would almost certainly mobilize significantly larger soil pressures. The same is also true for the mechanism indicated in Figure 5(b). Thus in both cases the RING strength predictions are likely to be quite conservative.

One way to address this issue properly is to move away from an indirect modelling strategy for the soil towards instead modelling the soil explicitly (i.e. using solid elements to represent the soil). Though this is in principle relatively straightforward, since RING is designed to be a rapid analysis tool an important challenge is to implement the capability in a computationally efficient manner. The requisite theoretical work is underway, with parallel experimental studies providing the necessary calibration data for cohesionless and cohesive fill materials. Whilst it is unlikely that sufficient validation will have been performed to enable explicit modelling of the soil to be implemented in RING 2.0, it is anticipated this will be incorporated in a subsequent release. Further discussion of the important issue of soil-structure interaction is provided elsewhere in the proceedings[xi].

4 OTHER PLANNED ENHANCEMENTS

In addition to the technical enhancements already described, feedback from users has indicated that the following enhancements should be implemented in RING version 2.0:

(i) Facility for individual, block/contact level, changes in material properties. The use of an object-oriented programming approach should make this easy to implement.

(ii) Customizable default values and inclusion of partial safety factors. This will be implemented in RING 2.0 by providing user-definable templates.

(iii) Improved documentation, including provision of a manual especially for modelling railway underline bridges.

(iv) Ability to save solutions as well as input data in a RING data file.

(v) More streamlined and easy to follow user interface, with less important details accessible only via ‘Advanced…’ or ‘More…’ buttons.

(vi) Addition of features commonly found in mainstream ‘Windows’ programs (e.g. ‘Undo’ functionality, context sensitive help etc).

In versions of RING beyond version 2.0 the object-oriented programming approach should facilitate easy introduction of further features, such as the explicit backfill elements already referred to and 3D modelling of bridges.

5 CONCLUSIONS

The RING masonry arch bridge analysis software has proved to be remarkably popular, being downloaded many thousands of times since its initial public release in 2001. Recent changes to the problem formulation used by RING, and outlined in the paper, have been shown to significantly increase its computational efficiency. Modelling friction using a non-associative rather than associative, frictional model is shown to make a modest (< 10%) difference in the predicted carrying capacity in the case of the multi-ring masonry arch bridge problems described in the paper. However, execution times are significantly extended (by a factor of up to 25×). It is intended that this model will be offered as an option in the next major release of RING (version 2.0, to be released in early 2005).

Soil-structure interaction in masonry arch bridges is an important area which is now the subject of active research at Sheffield University. It is demonstrated that the current soil model used in RING will tend to lead to over-conservative estimates of bridge strength when non-standard failure mechanisms are identified; this will be addressed in the future.

6 ACKNOWLEDGEMENTS

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