

A new serviceability approach to masonry arch bridge assessment

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ABSTRACT: The approach adopted in the paper utilises a Castigliano thinning analysis to investigate stress as a serviceability criteria and seeks, in the first instance, to determine the limiting value of stress which would, on average, produce a similar overall assessment performance. That is, bridges owners would get the same number of assessment failures as if based on ultimate load but the failing arches may have a different geometry and now the criteria for limit would be service based. This approach allows different materials/geometries to be identified, as likely to result in premature failure, but with the same overall probability of failure. It is considered that this more cautious approach will provide information which will, over time, assist bridge owners in starting to classify the types of bridges that are more at risk whilst at the same time not creating a hiatus in the process of bridge management. The paper details the approach and compares both assessment methods for a typical bridge.

1 INTRODUCTION

1.1 Introduction

Interest in masonry arch bridge assessment has gradually moved away from being primarily associated with the estimation of their current ultimate load capacity more towards concern that the longevity of this significant part of the infrastructure is not threatened by repeated overload. In order to properly develop serviceability based assessment methods there are two “ideal” approaches. The first is to undertake a wide scale investigation of the existing bridge stock and, assuming there are some loading based issues, to link this with the historic record of the loading that each bridge has endured, to develop a statistical “understanding” of the relevant issues. The second approach is to develop a really accurate (sophisticated) model of bridge behaviour at all loads up to ultimate and then to couple that with the historic record of loading but also with real new understanding of the long term behaviour of the principle bridge components (stone, mortar, soil) including durability, environmental damage, fatigue etc. The first approach requires a level of detail in bridge and loading records that is unlikely to exist and even if successful would not in itself allow developments of analytical based assessment methods. The second approach would require significant further development of the models of long term material behaviour which is unlikely to be forthcoming in the near future. A different approach has therefore been adopted for the current study where it is proposed to develop a serviceability based approach alongside ultimate load analysis such that bridge owners and their assessing engineers gradually gain more understanding of the pertinent issues.

In order to systematically identify differences between the MEXE, ultimate load approaches and serviceability based approaches it would be necessary to undertake assessments covering a wide range of geometric and material parameters. It was felt important that any new serviceability approach should initially return similar levels of bridge failures to the existing methods so that it did not create a hiatus in bridge management. It was therefore necessary to initially develop the statistical basis for the probability of occurrence of a range of arch parameters. It was accepted at the start that even with an analysis tool as easy to use as a spreadsheet¹, it would not be practicable, or even possible, to undertake sufficient assessments to properly cover the full range of all parameters using any, even slightly, sophisticated form of assessment tool. It was therefore decided to develop a statistical surrogate of the elastic cracking model, which retained the essential features but which could be included in a wide ranging parametric modelling study.

1.2 Serviceability assessment method development logical flow

As illustrated in Fig.1, the main development stages of this approach were therefore to:

- (1) Identify likely suitable candidates for a serviceability based approach.
- (2) Apply the serviceability assessment tool to a small sample of bridges and develop suitable overall parameter values that, on average, gave similar results to the Ultimate Limit State (ULS) assessment.
- (3) Apply these fixed serviceability parameter values to the analysis of a separate wider range of bridges using the same serviceability assessment tool.
- (4) Use these results to develop simple empirical models of the assessment tool, and of the ULS approach, and then to check their performance.
- (5) Apply the simple empirical models to the full range of bridge stock with the correct frequency of usage of any value appropriate to its probability of occurrence.
- (6) Modify the serviceability parameter values until the assessment methods give the same overall load assessment capacity as the ULS assessment methods.
- (7) Finally quantify types of arches that are more or less likely to be prone to serviceability failure than ULS.

2 STATISTICS

2.1 Introduction

100 real bridges were identified from books, papers, reports and the internet, and their basic geometric parameters determined including the span, rise, ring depth and crown fill depth. The basis for the development of the statistics of these bridge parameters are detailed below. Four parameters, span, rise, ring depth and crown fill depth, were studied although in this paper only a subsection of the parameters are detailed, just to show the statistical methods adopted. Cumulative probability functions were fitted using two models, a piecewise multilinear model and a hyperbolic tangent model. The probability density curves, needed for the sampling exercise, were then determined directly from the fitted cumulative probability curves. Finally the average and typical values are defined. In this paper only the span and rise statistics are detailed but a similar approach was adopted to determine the parameter values for arch ring depth and crown fill depth.

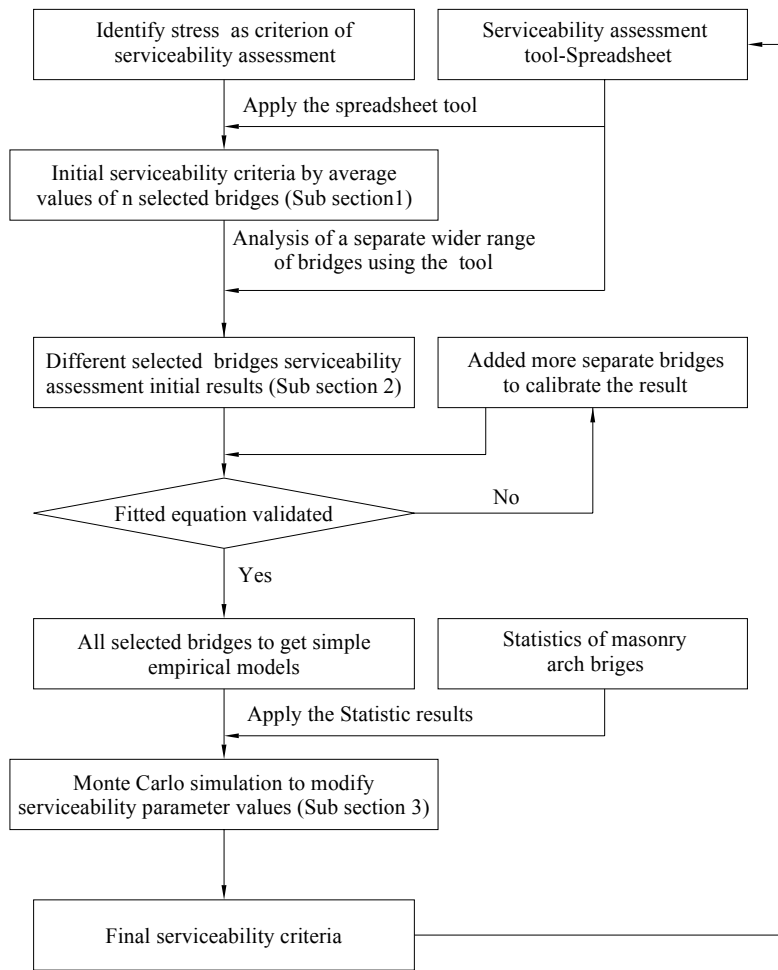
2.2 Analysis and results

The numbers of bridges found including just span is larger than the 100 for which all parameter values were determined; this is because many more bridges have only span values. In total the spans for 378 arches were recorded and their cumulative probability is detailed in Fig.2.

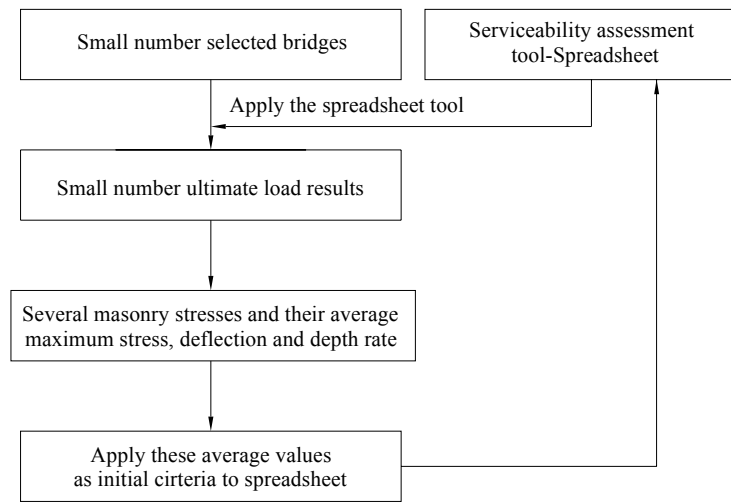
To undertake the regression for the multi-linear approach the spans were divided in to a number of groups and then linear regressions were applied to each group ensuring a continuous value at the group interface. The hyperbolic tangent function was selected as a suitable continuous function thought to be suitable for the full range and is given by equation (1).

$$y = \tanh(kx + b) \quad (1)$$

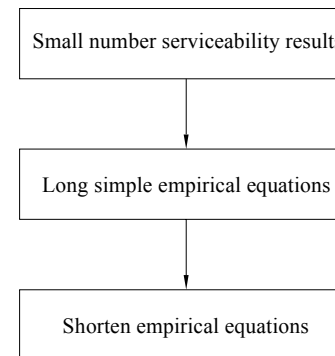
Where y is the cumulative probability of the span and x is the span, k and b are constants to be determined by the regression.



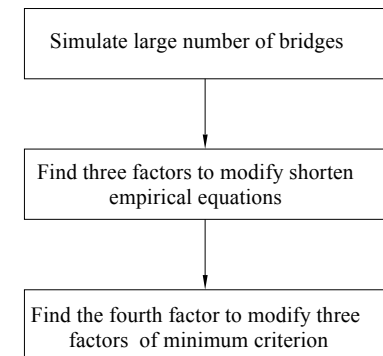
Serviceability assessment method development logical flow



Sub section 1



Sub section 2



Sub section 3

Figure 1: Serviceability assessment development flow diagram.

As illustrated in Fig.2, the multilinear line naturally looks closer to the real data, however the hyperbolic line is still a good fit and as its probability density is continuous, and the R^2 value of both fits are almost the same, the hyperbolic line was used to simulate the span distribution.

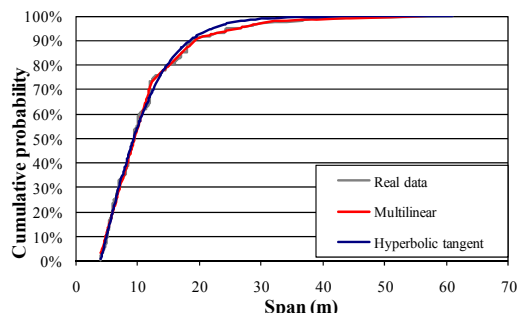


Figure 2: Cumulative probability distribution of the arch spans.

2.3 Statistical Outcomes

In determining the frequency distributions of the remaining geometric parameters it was decided that these generally needed to be linked to the dominant span parameter. In the subsequent sampling analysis it was assumed that there is no correlation between the different parameters and it was considered that this is closest achieved by using non-dimensional relative size values. The rise and the depth are therefore scaled to the span and the crown fill depth scaled to the arch ring depth, this seemed appropriate to the authors.

In fitting the cumulative probability distribution to the rise to span data it was poorly fitted by the hyperbolic equation and subsequently was modelled by the multi-linear approach. The best cumulative probability distribution model of the arch ring depth to span was the hyperbolic tangent curve and the best distribution model of the crown fill to arch ring depth was also the hyperbolic tangent curve.

The mean, median, mode and typical values of the bridge parameters are all listed in Table 1. The mode values are determined from the selected fitted curves. From Table 1, typical parameters are defined as a span of 10 metres, with a rise of 3.8m, arch ring depth of 0.53m and crown fill depth of 0.27m.

Table 1: Mean, median, mode and typical values of bridge parameters.

	L (m)	r/L	d/L	h/d
Mean	11.5	0.35	0.060	0.61
Median	9.5	0.36	0.053	0.50
Mode	4.0	0.50	0.053	0.14
Typical Value	10.0	0.38	0.053	0.50

In determining the probability statistics for the general population of arch bridges there are undoubtedly national and even regional variations. Some of the sources of information will also not be representative of the actual population, for example long span bridges are often famous (e.g. Pontypridd) and are likely overrepresented in many sources. This was one of the reasons for adopting the hyperbolic tangent line which in Fig.2 slightly under represents this particular sector.

3 CHOISE OF STRESS CRITERION

3.1 Introduction

Following detailed consideration of the stresses developed both with moving load patterns and loading to limit state (ultimate limit or serviceability limit) at the critical load location, using the spreadsheet, the relevant serviceability stress was defined as the maximum intrados or extrados

stress occurring anywhere in the arch ring, except at the abutment. Earlier work^{2,3} had limited stress consideration to the intrados incipient hinge area between the applied load and the abutment remote from the applied load on the basis that the extreme extrados stresses under the applied load were confined by the load above to such an extent that it was not considered a critical location. The far abutment hinge area was not considered critical as this is, almost universally, a poorly defined support with there frequently being backing masonry material in this area that effectively extends the support vertically, such that the extreme extrados stresses predicted by the analysis in this area would not, in reality, develop. This second consideration was accepted in the present study but the extrados area under the applied load was considered legitimate as the confining stresses were investigated in the present study and were generally considered to be too low to provide enhanced confining support.

3.2 Increasing load to limit stress in critical positions

Fig.3 displays the intrados and extrados stress along one of the sample arches (C26) when loaded near the quarter point, also displayed is the effective arch thickness, zeroed to the original intrados position. As illustrated in the figure the maximum stress normally occurs under the load position with the peak stresses reducing with the distance of the incipient hinges from the live loading. This stress figure is for a bridge under a service load but the ultimate load stress figure is of a similar shape. Graphs similar to this allow the load, at which the maximum stress reaches a specified value, to be accurately determined.

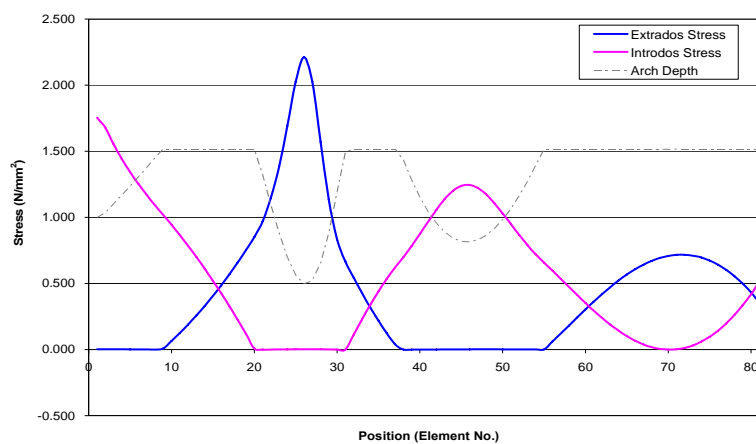


Figure 3: Intrados and Extrados stress in Bridge C26

3.3 Moving load over the whole bridge

Fig.4 displays the variation of the maximum stress, either Extrados or Intrados, achieved by a specific live load moved over arch bridge C26 in a number of increments. As illustrated, the critical service load position for a normal masonry arch bridge is near the quarter point, similar to the ultimate load critical position. It is also apparent that whilst the peak in Fig.3 at the location of the load is quite sharp, the variation of that peak with location, as demonstrated in Fig.4, has a significant plateau. The identification of the critical load position in a serviceability based analysis only therefore needs to be approximate.

4 INITIAL STRESS CRITERIA DETERMINATION

In this section, as illustrated in sub section 1 of Fig.1, a small number of bridge geometries were initially selected, each coupled with a number of different assumed masonry material ultimate stresses. These bridges were then analysed, using the spreadsheet based cracking elastic analysis¹, to determine their ultimate load capacities. From these ultimate load results the standard assumption that damage starts to occur at 50% of the ultimate load was applied in

order to determine initial estimates of the maximum allowable serviceability stresses for the bridges. These results were divided into different groups according to their assumed ultimate allowable masonry stresses. The average values of actual serviceability stresses of these groups were then used to define the initial serviceability stress based criterion. For example, when the ultimate stress was 5 N/mm², then for each bridge at this material stress, the ultimate loads are calculated at their individual critical position first, and then one half of that ultimate load was moved across that bridge in a number of steps and the maximum stress values recorded. This was repeated for each of the nine bridges and at the four ultimate stress values. Finally the average of the stresses was determined as the initial stress based serviceability criterion. The results of this exercise are tabulated in Table 2 which shows the critical load position stresses for each of the four assumed stress categories. Some simulations were undertaken at this stage to investigate the difference between the use of quarter point and critical loads for both ULS and serviceability based criterion and the results indicated that both approaches produced similar results although the remaining work always used the most critical load positions.

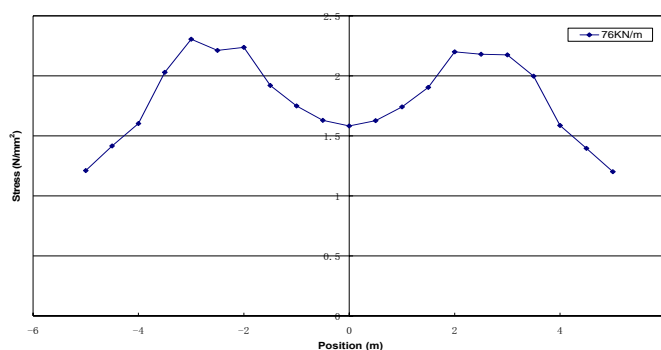


Figure 4 : Variation of peak stress with load position in Bridge C26

Table 2 : Initial serviceability criterion

Ultimate Stress (N/mm ²)	Critical Point Serviceability Stress (N/mm ²)
5	2.05
10	2.69
15	2.89
20	3.05

The variation of serviceability stress with ultimate stress contained in Table 2 should allow linear interpolation to be accurately used within the range of tabulated values. The next stage is then to develop simple empirical models of arch behaviour.

5 SELECTED BRIDGES INITIAL SERVICEABILITY RESULTS

In Sub-section 2 of Figure 1 a sample number of bridges was initially selected and analysed in order to develop simple empirical equations for both ULS and SLS. Initially one group of bridges was selected randomly from the bridge probability densities, as developed above in Section 2. A second group of the same size was added, these were made up of the first group of bridges but for each bridge one parameter value was varied significantly, so that there was a wide range of values for every parameter. The size of the sample was increased in stages, as shown in Fig.1, until the coefficients in the equations, detailed below in section 6, stabilised. The stress based serviceability criteria detailed in Table 1 were utilised with the spreadsheet analysis to determine the serviceability load capacities.

6 SIMPLE EMPIRICAL MODELS

Simple empirical functions for the ULS and SLS were developed in terms of the arch span, rise, ring depth, crown fill depth, fill material Rankine passive coefficient, density of arch and fill material, modulus of arch and fill material, and arch material stress. The functions were developed from a multi-linear regression of the elastic cracking spreadsheet results for the bridges detailed in section 5.

Following some detailed consideration of the equation coefficients, and the statistical significance of each to the accuracy of the estimate, reduced forms of the equation were developed that appropriately captured the effect of the geometric parameters. For the ULS the reduced (shortened) form of the regression equation is given as Equation (2)

$$P_u = 294 \cdot L^{1.73} \left(\frac{h}{d} + 1\right) \left(\frac{d}{L}\right)^{2.06} \quad (\text{Tonnes}/2.5\text{m}) \quad (2)$$

7 SIMULATIONS TO MODIFY SERVICEABILITY PARAMETER VALUES

Monte Carlo simulation was then used, as detailed in Fig.1, to simulate a large number of bridges made up of different spans, rise, ring depth and fill material depth each selected in probability to its occurrence, as previously determined. The form of the shortened version of SLS equation was modified until the modal values of the results for equation (2) and that for the SLS were the same. The resulting simple (shortened) empirical SLS equation is given as equation (3).

$$P_s = 230 \cdot L^{1.27} \left(\frac{h}{d} + 1\right)^{0.527} \left(\frac{d}{L}\right)^{1.8} \quad (\text{Tonnes}/2.5\text{m}) \quad (3)$$

Application of equations (2) and (3) to a large number of bridges will result in 50% of the bridges having a higher ULS load capacity and 50% a higher SLS capacity and, as the sample bridges have been selected in proportion to their occurrence, on average a bridge owner should get the same overall level of assessment limit failures from both equations.

As equation (3) is no longer the result of a multi-linear regression of the results from sections 5 and 6, it was necessary to modify the initial serviceability stress criterion in Table 2. The values in Table 2 were systematically adjusted until the application of the process of sections 5 and 6 resulted in a solution that both had a modal value which balanced the ULS simulations and was itself the regressed solution of the sample bridges. This leads to the final serviceability criterion given in Table 3.

Table 3 : Final serviceability criterion

Ultimate Stress (N/mm ²)	Critical Serviceability Stress (N/mm ²)
5	2.18
10	2.87
15	3.08
20	3.25

7 COMPARISON OF SERVICEABILITY WITH ULTIMATE ASSESSMENT RESULTS

A comparison of the ULS and SLS results for a typical bridge with one parameter changed for each analysis is shown in Fig.5 for the effect of span and rise. The results show the similarity of ULS and the stress based SLS simulations for this bridge.

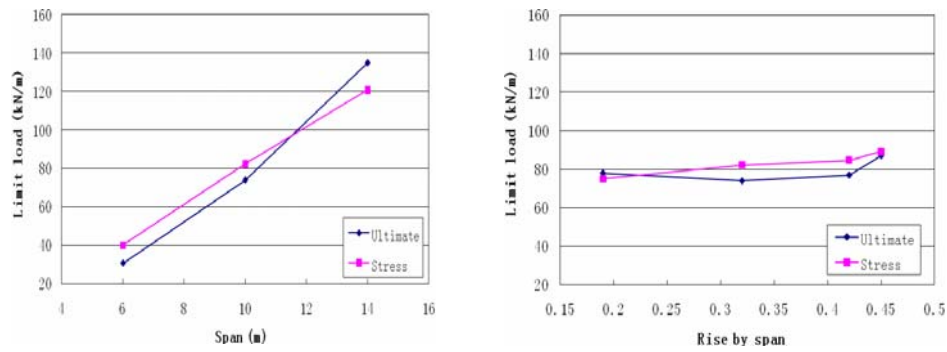


Figure 5: Variation of a typical arch ULS and SLS with span and rise to span ratio.

8 CONCLUSIONS

A stress based serviceability assessment method is fully developed in this paper but the method adopted is equally appropriate for a deflection based approach. The SLS method has been developed specifically to result in similar overall success rates to ULS in load assessment.

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