Limit behaviour of partially reinforced masonry arch bridges

G. Buffarini
ENEA Casaccia Research Centre,Rome, Italy

P. Clemente
ENEA Casaccia Research Centre,Rome, Italy

G. de Felice
University Roma Tre, Rome, Italy

A. Satta
University Roma Tre, Rome, Italy

ABSTRACT: The mechanism model gives a suitable interpretation of the limit behaviour of masonry arches that allows to assess the safety condition according to geometrical characteristics rather than material strength. Different issues must be faced when considering reinforced arches, since crushing and sliding may occur due to the increase in bending capacity provided by the reinforcement. For partially reinforced arches, a simplified approach based on limit analysis can be used, in which the failure condition according to the hinge mechanism is first checked and the strength condition of masonry in compression is then verified. Referring to a single span bridge under concentrated travelling load, the effectiveness of a partial reinforcement with varying length, either at the intrados or at the extrados of the arch, has been analysed; the resulting diagrams expressed as a function of load position can be used for a preliminary design of the reinforcement.

1 INTRODUCTION

The interest in masonry arches lies in the assessment of effective load carrying capacity of structures which are still in use; think for instance to existing railway arch bridges, which have to comply with current revisions and updating of loading conditions.

A suitable interpretation of the limit behaviour of masonry arches is given by the mechanism model, in which failure occurs only when a sufficient number of “hinges” forms to turn the structure into a mechanism. The formation of hinges can be inhibited by means of reinforcements; in fact, when the reinforcement is applied, the behaviour of the arch changes significantly, since the reinforcement acts as a constraint for a class of mechanisms by increasing, at the same time, the compression stress in the cross-section. Therefore, masonry compression strength can no longer be considered as infinite and crushing in compression must be checked too. In the presence of reinforcement, the trust line is allowed to lie out of the cross section and assume a high slope with respect to the centre line of the arch; as a result, sliding between voussoirs may occur and must be checked as well. Besides, ripping or debonding of the reinforcement must be checked too.

Among the strengthening techniques, the use of fibre reinforced polymers (FRP) appears one of the most promising methods; the application of reinforcement may strongly increase the load carrying capacity of masonry arches; however, the appraisal of the new aforementioned collapse mechanisms, which may occur with limited ductility, is one of the main drawbacks of this technique.

A comprehensive investigation of the structural behaviour under incoming loads has been the subject of a previous paper (Buffarini et al., 2006), which investigates the effects on medium to long-span bridges subjected to vehicular load.

In the present paper, a methodology for the design of the reinforcement of masonry arch bridges is presented. After a recall of the limit state conditions of reinforced sections, the limit behaviour of reinforced arches is assessed and the increase in load carrying capacity due to application of the reinforcement, of variable length, either at the intrados or at the extrados of the arch, is analysed for single-span arch bridges with fixed abutments, under concentrated travelling load.

2 COLLAPSE OF THE REINFORCED SECTION

The application of plastic theory to masonry is based on the well-known three hypotheses on material behaviour: a) joints between voussoirs have no tensile strength, due to the absence or to the weakness of mortar between masonry blocks; b) compression strength is assumed to be infinite, since stresses in service conditions are low enough to avoid crushing of the material; c) sliding failure cannot occur, since friction between voussoirs is high enough. For reinforced sections the two hypothesis b) and c) about the strength in compression and the shear forces are no longer valid, since crushing and sliding may occur due to the increase in bending capacity provided by the reinforcement. The limit state conditions of reinforced cross-section were analyzed in details by Triantafillou (1998), Faccio et al. (2000) and Foraboschi (2001, 2004).

Referring to a rectangular cross-section of width b and height t, let us assume the reinforcement to be non-active until the line of thrust lies into a central portion of the cross-section (Fig. 1). In more details, if the eccentricity $e \le t/3$, then the section behaves like the non-strengthened section and only compression stresses are needed for the equilibrium. Only if $e > t/3$, the reinforcement contribution is required for the equilibrium at the edge where the crack forms. In this case, the tensile force T that develops in the reinforcement is balanced by an increase in the compressive force C in the cross section at the edge opposite to the reinforcement:

$$C = N_d + T \tag{1}$$

2.1 *Crushing of masonry in compression*

According to experimental evidences (Foraboschi, 2004), we assume the resultant compression force C to be at a distance $t/3$ from the centre (Figure 1). In order to avoid crushing of masonry that may occur in reinforced cross-sections, the compression C should not exceed the limit compression C_u given by

$$C_u = f_m\, bt/3 \tag{2}$$

where f_m is the strength of masonry in compression, b is the total width of the arch. As well known, the effective width of the cross section, which absorbs the increase in compression, depends on the reinforcement details (number, size and position), on masonry work and on the level of axial forces. We suppose the reinforcement to be deployed over the whole arch width, so we assume the total width b of the arch as effective. The ultimate bending moment after which crushing takes place is therefore:

$$M_u = \frac{5}{6} \cdot C_u t - \frac{1}{2} \cdot N_d t \tag{3}$$

2.2 *Shear failure*

The shear forces are usually very low in non reinforced arches, they can become high in reinforced arches when the line of thrust lies outside of the cross section and assume a high slope with respect to the arch profile. It is worth noting that if the resultant is external of the cross section, then the compression force C increases and shear resistance increases too. To avoid shear failure, the shear V_d due to external load should not exceed the ultimate shear V_u given by:

$$V_u = \mu\, C \tag{4}$$

where μ is the friction coefficient of masonry.

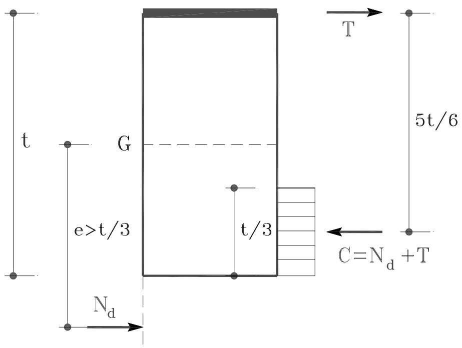


Fig. 1 : Equilibrium of the reinforced cross-section.

2.3 Failure of the reinforcement

Collapse of the reinforcement may occur either to debonding or to tensile failure of reinforcement. As a result of the arch curvature, when the reinforcement is in tension not only shear stress, but also normal stresses develop at the interface. It can be shown (Foraboschi 2004) that if the reinforcement is properly applied, the cracks develop in masonry rather than in epoxy matrix, with the ripping of a thin layer of brickwork. The ultimate traction in the strips to prevent ripping is given by:

$$T_{bu} = n\, f_{mt}\, w\, R \tag{5}$$

where n is the number of reinforcement strips of width w, R the radius of curvature of the arch and f_{mt} the tensile strength of brickwork.

Finally, denoting by f_t the tensile resistance of reinforcement per unit width of strip, then the limit traction in strips to prevent tensile failure is given by:

$$T_{tu} = n\, w f_t \tag{6}$$

If we put

$$T_u = \max\left(T_{bu}, T_{tu}\right) \tag{7}$$

the corresponding ultimate bending moment is:

$$M_{tu} = \frac{5}{6} T_u\, t + \frac{1}{3} N_d\, t \tag{8}$$

3 LIMIT ANALYSIS OF REINFORCED ARCHES

A suitable interpretation of the limit behavior of masonry arches is given by the mechanism model, based on the theoretical and also experimental studies of the eighteen and nineteen centuries, in which failure occurs only when a sufficient number of “hinges” forms to turn the structure into a mechanism. The so-called “mechanism method” was proposed in its present form by Heyman (1966, 1969). Collapse must be viewed as a geometrical problem rather than as a problem of strength of materials; failure of the arch is not related to crushing of masonry, but only to its shape.

With these hypotheses the limit analysis was applied to masonry arch bridges, by using the principle of virtual work to express the equilibrium condition (Clemente et al. 1995). This allowed to point out the main features of masonry arch bridge behaviour and the influence of the geometrical and loading parameters on it, both under vertical and horizontal loads (Clemente and Raithel 1998).

The application of a reinforcement at the intrados or at the extrados increases the strength of the arch. In fact, some collapse mechanisms are restraint, since opening of the joints on the reinforcement side is not permitted, and the line of trust is allowed to lie out of the arch profile at the opposite edge of reinforcement. However, as a consequence, the local failure modes described in the previous paragraph become possible and have to be checked.

The same procedure used for limit analysis of non reinforced masonry arches can be applied to partially reinforced arches but, in this case, the set of failure mechanisms to be investigated according to the kinematic approach, is only a subset of the set of failure mechanisms of the non-reinforced arch. Therefore, the search for the minimum load factor in the subset of mechanisms allowed by the reinforcement, will give rise to a collapse load multiplier not lower than that of non-reinforced arch.

Once the effective collapse mechanism is found, the corresponding line of trust can be traced (Fig. 2), and the reinforcement can be designed in order to satisfy the conditions on local strength. If the strength is fulfilled with the desired safety factor, according to limit analysis, this ensure that the load factor is the one obtained and the failure may develop with the classical hinge mechanism. The reinforcement designed according to this procedure ensures ductility similar to that of non reinforced arch together with an increase in load carrying capacity.

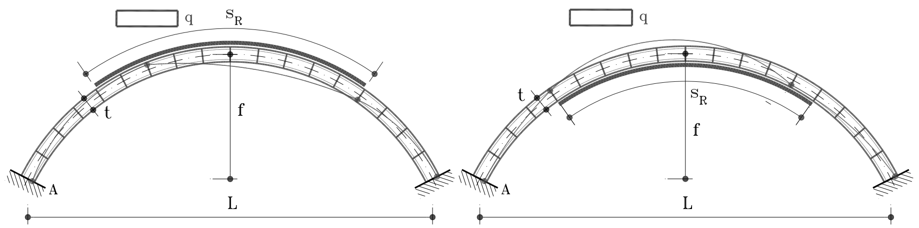


Fig. 2 : Hinge locations and thrust line for FRP at the extrados and at the intrados.

4 LIMIT BEHAVIOR OF REINFORCED ARCHES

The procedure previously described, has been applied to the arch bridge having a span $L = 15.60$ *m*, a sag-ratio $f/L = 0.25$ and a depth-to-span ratio $t/L = 0.05$, subject to the dead load g_0 and g_1, and to a travelling force $F = 100$ *kN*. The force acts as a load uniformly distributed over a length *a* as shown in Fig. 3, where $\beta = 40°$. The weight per unit volume of the arch is supposed to be $\gamma_0 = 22\,kN/m^3$, while the backfill, which has the height $h = 0.75\,m$ above the extrados of the crown, has a weight per unit volume $\gamma_1 = \gamma_0$. The arch has been discretised in a high number of voussoirs and the analysis has been carried out by using a computer code set up on purpose. A maximum value of eccentricity equal to *t/3* was assumed in the analyses of non reinforced sections, in order to account for a limited strength of the material.

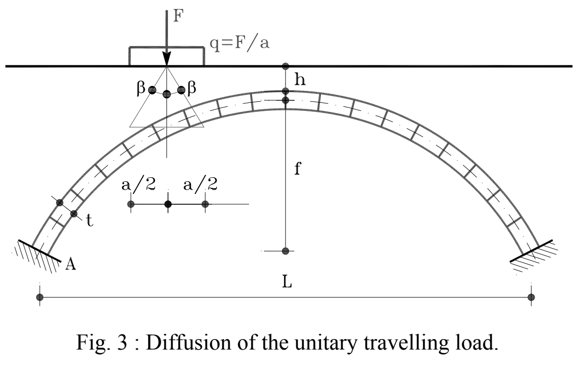
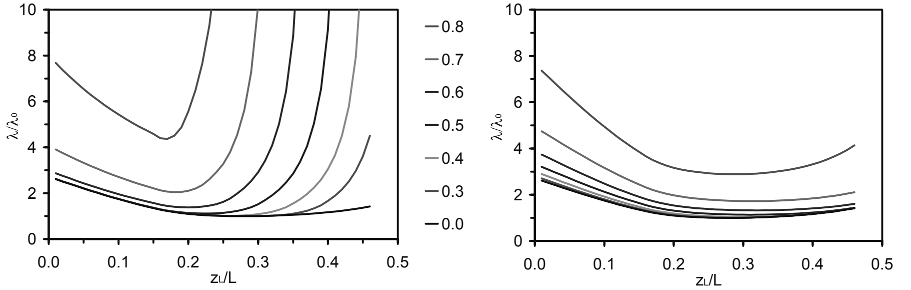


Fig. 3 : Diffusion of the unitary travelling load.

In absence of reinforcement, the minimum load factor is equal to 3.00; the collapse mechanism develops with hinges A and D at springing; hinge B forms at $z/L = 0.25 \div 0.45$ when the load position ranges from zero to 0.5, while hinge C ranges between $z/L = 0.60 \div 0.75$. In the following the values of all the parameters are put in the non dimensional form by dividing them for the corresponding values relative to the case of the minimum load factor of the non reinforced arch, which is characterized by the subscript "0".

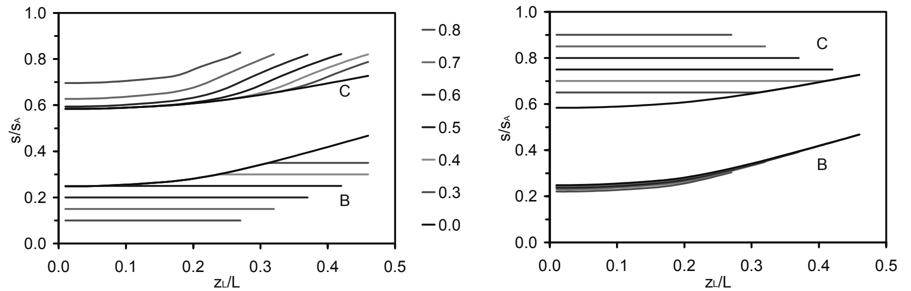
Two different deployments of the reinforcement have been considered: reinforcement at the extrados (type 1) and reinforcement at the intrados (type 2). The strips are supposed to start from the section at crown, and to have an increasing extension up to the sections at $s/s_A = \pm 0.40$, s being the curvilinear abscissa from the crown and s_A the total length of the arch centre line.

In Figs 4a) and 4b) the load bearing capacity of the bridge, expressed by the load factor λ is plotted against z_L/L, for different values of the ratio s_R/s_A between the reinforcement length and the total length of the arch centre line, in the two cases of extrados and intrados deployments. In the first case the presence of the reinforcement influences very much both the value of the minimum load factor and the corresponding load position. When the reinforcement is at the extrados the load factor increases with s_R but the load condition does not change. The position of the hinges are reported in figures 5a) and 5b). The influence of the reinforcement is evident.



a) b)

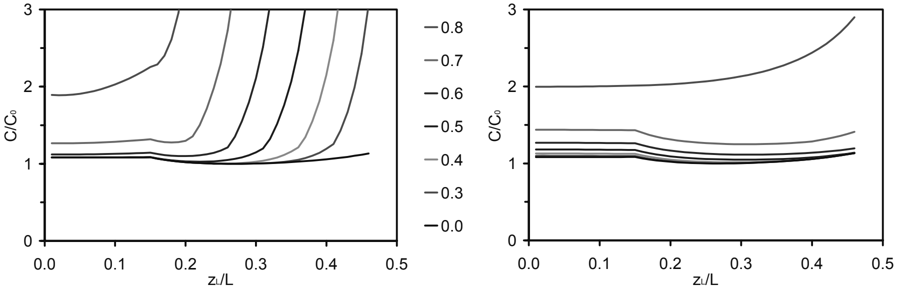
Fig. 4 : Load factor versus z_L / L for different ratios s_R/s_A (reinforcement at the a) intrados, b) extrados.



a) b)

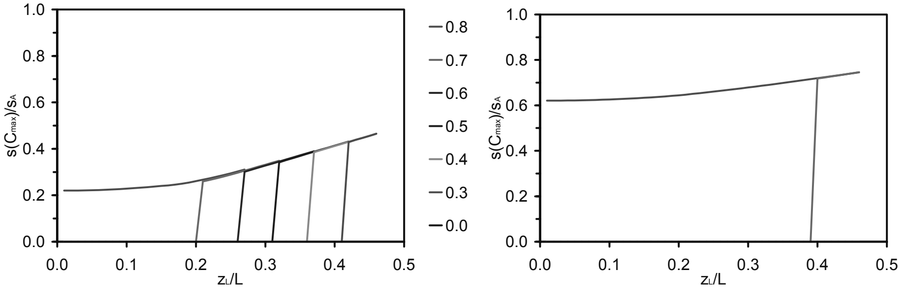
Fig. 5 : Load factor versus z_L / L for different ratios s_R/s_A (reinforcement at the a) intrados, b) extrados.

The maximum values of the compression force C is plotted in Figs. 6a) and 6b), normalized with respect to the maximum compression C_0 of the non-reinforced arch. The extension of the reinforcement has the same effects in both cases but the load position has a stronger influence in the case of reinforcement at the intrados. The section in which C is maximum is plotted in Figs. 7a) and 7b). The compression force C is maximum at springing when z_L/L is low, and move to a section in the reinforced zone (between 0.2-0.4L for the reinforcement at the intrados and between 0.6-0.7L for the reinforcement at the extrados) if z_L/L gets higher. The load condition, which separates the two behaviors, depends on s_R/s_A. In particular, in the case of reinforcement at the extrados, for $s_R/s_A < 0.6$, C is always maximum at the springing, independently of the load condition. For $s_R/s_A > 0.8$, C is maximum always at $s/s_A = 0.6 \div 0.7$. For $0.6 < s_R/s_A < 0.8$, the section in which C is maximum depends on the load position.



a) b)

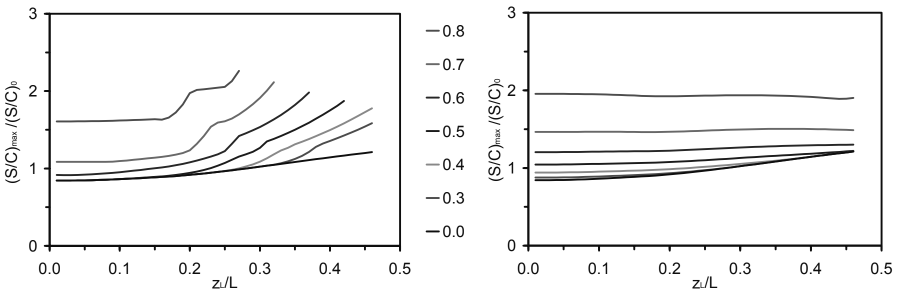
Fig. 6 : Maximum axial force for different ratios s_R/s_A (reinforcement at the a) intrados, b) extrados.



a) b)

Fig. 7 : Position of C_{max} for different ratios s_R/s_A (reinforcement at the a) intrados, b) extrados

The maximum values of the ratio *S/C* between the shear and the compression forces, normalized with respect to the maximum ratio $(S/C)_0$ of the non-reinforced arch, are plotted in Figs. 8a) and 8b). Finally, the maximum values of the tension in the reinforcement, normalized with respect to the maximum compression C_0 of the non-reinforced arch, are plotted in Figs. 9a) and 9b).

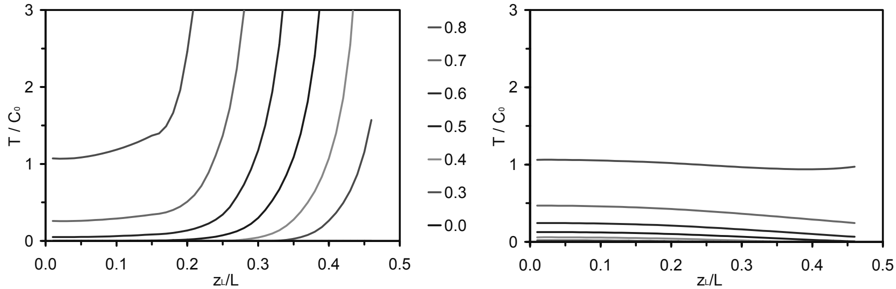


a) b)

Fig. 8 : Maximum ratio *S/C* for different ratios s_R/s_A (reinforcement at the a) intrados, b) extrados.

In Fig. 10, the minimum load factor is plotted against the reinforcement length, showing that reinforcement application succeed in increasing the load carrying capacity only for extensions. $s_R/s_A > 0.5$.

In Figs. 11a) and 11b), the maximum values of compression and the maximum values of tension in the reinforcement obtained for the load position associated to the minimum load factor are plotted versus s_R/s_A. These diagrams are very useful for the masonry check and the design of the reinforcement.



a) b)

Fig. 9 : Massimum tension force for different ratios s_R/s_A (reinforcement at the a) intrados, b) extrados.

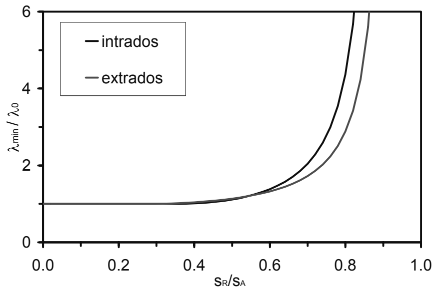
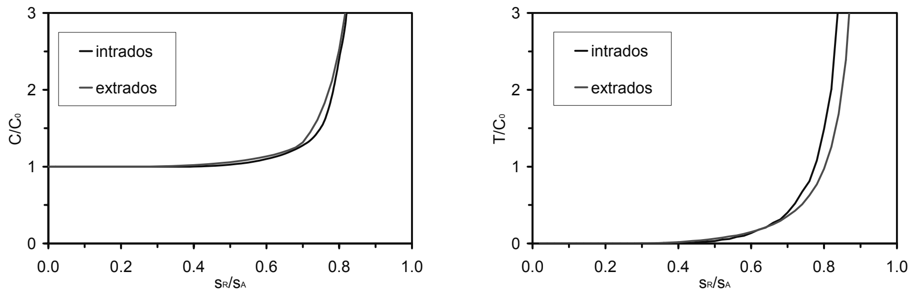


Fig. 10 : Minimum load factor λ_{min}/λ_0 for different ratios s_R/s_A.



a) b)

Fig. 11 : Maximum ratios a) C/C_0 and b) T/C_0 against s_R/s_A.

5 CONCLUSIONS

In this paper, the increase in load carrying capacity of arch bridges due to the application of reinforcements of variable length, either at the intrados or at the extrados, is analysed. According to the limit analysis kinematic approach, the ultimate load is obtained searching for the minimum in the subset of mechanisms allowed by the reinforcement. Beside, the conditions on local strength, to avoid failure of masonry in shear or in compression, and debonding are verified. Single span bridges under concentrated travelling load have been considered, and the effectiveness of a partial reinforcement has been pointed out. The results, clearly show the dependence of the load carrying capacity on the length of the reinforcement; in the present case, as expected, a partial reinforcement with length $s_R/s_A<0.6$ does not provide any increase in the capacity; however this is not the case of multi-span bridges, where an even short reinforcement in the central part of the span, is expected to increase the capacity by restraining the collapse mechanisms that involve the piers.

The resulting diagrams not only explain some features of the behaviour of reinforced arches but also provide a tool for preliminary design of the reinforcement, under condition that the arch, when subjected to increasing travelling load, fails according to the hinges mechanism.

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