Structural capacity of masonry arch bridges to horizontal loads

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ABSTRACT: This work deals with the seismic vulnerability evaluation of existing masonry arch bridges. A parametric study has been carried out to determinate how the capacity curve of the arch changes with its geometric configuration, in the hypothesis of infinitely rigid abutments. The capacity curve is obtained by means of a limit analysis approach. The study is aimed to identify on one hand the horizontal load multiplier that activates the kinematic mechanism, on the other hand the collapse displacement. The numerical results of the structural capacity so found have been compared with the results available in literature.
In parallel, the structural capacity of arch bridges with span up to 1.5m has been evaluated. In this case, also the abutments participate to the kinematic mechanism. After identifying the most probable collapse mechanism, a parametric study has been carried out to evidence the influence of passive earth pressure on the capacity curve trend.

1 INTRODUCTION

This study has been carried out in the framework of the collaboration between the Department of Structural and Transportation Engineering of the University of Padua and the Direction of Italian Railway (R.F.I.) at Mestre Compartment (Italy). It is aimed to the seismic vulnerability analysis of the masonry arch bridges on the railway line Bologna-Padua. Due to the numerous constructions to be analyzed, a simple and quick automatic procedure has been implemented. In fact, it is sufficient to have only few structural geometry parameters in order to carry out the structural capacity evaluation of masonry arch bridges.

This work describes the procedure finalized to find the ground acceleration that activates the arch mechanism and the ultimate displacement at collapse of one control point of the structure.

The approach here described is based on the kinematic analysis through the application of the principle of virtual work, in the two hypotheses that the abutments participate or do not participate into the kinematic mechanism. The masonry tensional state has not been taken into account. In fact, as can be found in literature, the tensional level in masonry arches is not significant for the safety assessment and the resulting deformation is not relevant. The collapse mechanism activation, which depends on the arch geometry and on the applied loads, has, instead, an influence on the arch capacity. The mechanism activation, due to the horizontal load, is characterized by the formation of three rigid voussoirs and four hinges, which are situated in points where the thrust line crosses the masonry of the arch ring (Clemente, 1998).

For the analysis of bridges that do not have infinitely rigid abutments, a different procedure has been implemented, which takes into consideration the participation of the abutments into the collapse mechanism. Also for this collapse modality, the capacity curve of the structure and the ultimate displacement at collapse can be obtained. In the case of abutments built against-earth, the study of the influence of the passive earth thrust that causes a change in the capacity curve of the structure is of particular interest.

2 KINEMATIC MECHANISMS IMPLEMENTED IN THE AUTOMATIC PROCEDURE

The procedure here presented consists in the application of the principle of virtual work and can be applied to the analysis of circular, parabolic and generic shape masonry arches.

For the kinematic analysis execution, the arch is discretized into rigid voussoirs (figure 1), whose centroid virtual displacement is determined. The filling material is exclusively taken into account as applied mass and divided into a number of voussoirs equal to the number of arch voussoirs. The passive thrust that the fill creates is disregarded. This hypothesis is justified by the relatively low value of the arch rise, which entails the mobilization of small earth wedges close to the arch extrados and springings (figure 1). Anyway, neglecting this stabilizing thrust is on the safety side.

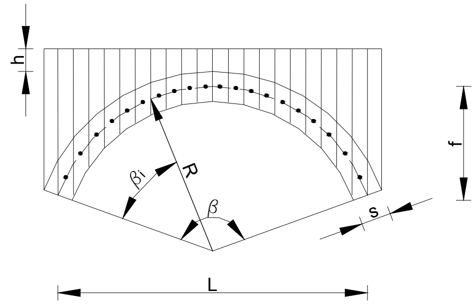

Figure 1: Voussoirs schematization of the arch.

The following procedure aims to provide for:

- the ground acceleration that activates the mechanism;
- the ultimate displacement at collapse of one control point of the structure (for example, the keystone centroid).

The assumed hypotheses are:

- absence of sliding between voussoirs (Heyman, 1982);
- infinite compressive strength of masonry (Heyman,1982);
- infinitely rigid arch abutments;
- large displacements (for the evaluation of the ultimate displacement at collapse).

The third hypothesis is connected to the structural type taken into account. A substantially different kinematic mechanism is activated in the case of slender and tall abutments that participate to the mechanism, for which a specific procedure, illustrated in section 5, has been implemented. The hypothesis of large displacements is fundamental to evaluate the collapse displacement, because this problem is affected by second order effects.

The evaluation of the arch structural capacity is articulated into the following steps:

- determination of the collapse mechanism (Heyman, 1982; Clemente, 1998);
- application of the principle of virtual work to the original arch configuration, for the determination of the ground acceleration that activates the collapse mechanism. This is represented by the horizontal load multiplier α and is the first point of the capacity curve (figure 2);
- application of the principle of virtual work to the different deformed arch configurations, in order to determine the ultimate displacement d_u of the capacity curve (figure 2).

The determination of the hinges location at the activation of the mechanism (Clemente, 1998), is an iterative approach based on the principle of virtual work. In case of arches supported by infinitely rigid abutments and in absence of fill, Clemente (1998) provides for graphic shape results about hinges configuration. In case of abutments which participate to the mechanism, the problem can be treated using a finite element code, but this aspect will be analyzed in paragraph 5.

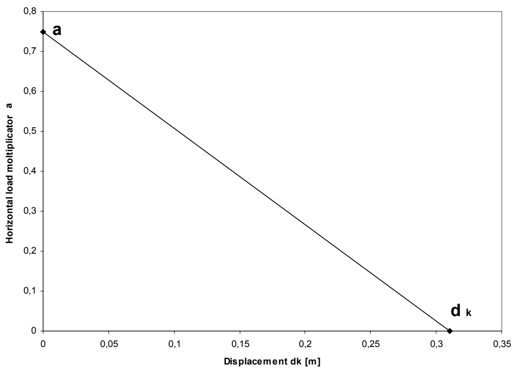

Figure 2: Example of capacity curve with the characteristic points α and du

The next step is the application of the principle of virtual work to the original indeformed arch shape. The equation of the principle of virtual work is the following:

$$\alpha \sum_i P_i \cdot \delta_{x,i} - \sum_i P_i \cdot \delta_{y,i} - \sum_j F_j \cdot \delta_j = 0 \tag{1}$$

where α is the horizontal load multiplier that activates the mechanism, P_i represents the weight force of the arch voussoirs and of the filling voussoirs, F_j refers to the generic external force applied to the structure, δ stands for the virtual displacement of the load application point.

The last step consists in the evaluation of the ultimate collapse displacement d_u. The centroid of the keystone has been chosen as control point. The principle of virtual work is now applied to a generic deformed shape of the structure, as far as the configuration that makes null the α multiplier is attained. In other words, the procedure is repeated until the arch configuration does not have any residual capacity to the seismic action. In this case, the deformed shape of the structure represents the unknown value. The equation (1) becomes:

$$\sum_i P_i \cdot \delta_{y,i}(\varphi) + \sum_j F_j \cdot \delta_j(\varphi) = 0 \tag{2}$$

where φ represents the finite rotation of the structure and identifies the unknown deformed configuration. From the resolution of the equation (2), the ultimate displacement at collapse d_u can be determined. Thus, the capacity curve is univocally defined.

For the application of the equation (2), it is necessary to know the displacement of the application point of each force, of the weight force of the arch voussoirs and of the filling voussoirs and the external applied force. Until the geometric problem is in a small displacement domain, it is simply solvable. In the large displacements domain, the position of rotation centre O has to be updated while the mechanism is evolving (figure 3). It can be seen in figure 3 that the centre O moves itself to the position O'.

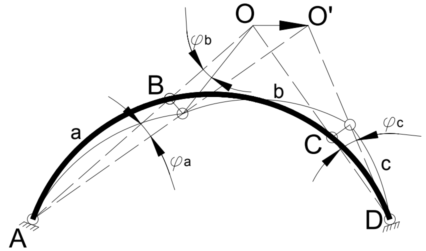

Figure 3: Kinematic mechanism implemented in the procedure.

The last aspect to be highlighted in the implemented procedure, is the possibility of schematizing the inertial action of the filling material with four different models as in (Clemente, 1998).

3 COMPARISON WITH LITERATURE RESULTS

A comparison with the numerical results reported in (Clemente, 1998) has been done, in order to assess the results obtained by means of the implemented procedure. A good agreement between the results has been obtained. The comparison parameters were:

- the horizontal load multiplier α that activates the mechanism;
- the ultimate rotation φ_v at collapse for the arch section *a*.

The results of the comparison are reported in figure 4. In the diagram, the axis of abscissas gives the nondimensional ratio thickness/radius of the arch, while the axis of ordinate gives the horizontal load multiplier α, which activates the mechanism (figure 4a), and the ultimate collapse rotation φ_v (figure 4b). Each curve is connected with a given centre angle β (figure 1). The obtained curves are situated above those given by (Clemente, 1998), which have been calculated neglecting the presence of the filling material, and thus represent only a limit value for this study. Actually, in the analyses carried out by the authors the filling material has been taken into account as applied mass, and this condition provokes an increase of the ground acceleration that activates the collapse mechanism and of the related collapse rotation.

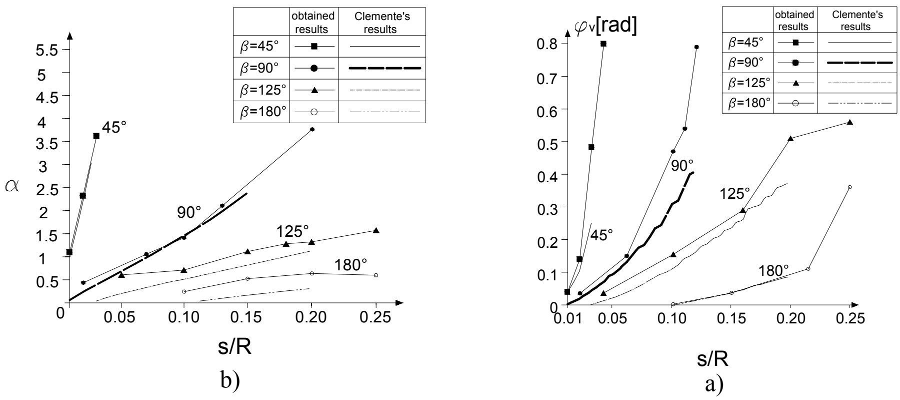

Figure 4 : Comparison between the horizontal load multiplier that activates the mechanism (a) and the collapse rotation (b) obtained in (Clemente, 1998), and with the implemented procedure (with filling material).

From the study of the collapse rotation, two arch collapse modalities emerge:

- achievement of the equilibrium condition under vertical loads (attainment of the condition that makes null the α multiplier);
- achievement of the condition of unstable equilibrium.

Collapse due to the achievement of the equilibrium condition under vertical loads is recurrent in round arches or similar, while collapse due to instability is recurrent in depressed-arches. The second collapse modality is possible if the alignment of hinges A, B, and C is reached, as can be seen in figure 5.

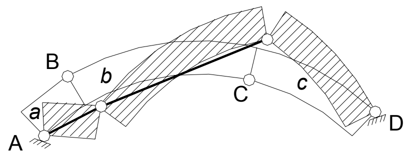

Figure 5: Unstable arch configuration due to the alignment of the three hinges A, B and C.

In fact, for major rotations the two sections *a* and *b* sway down and the arch collapses.

For round arches or similar, the alignment condition for the hinges is reached through a rotation of the arch sections that is bigger than the rotation requested by depressed-arches (figure 6).

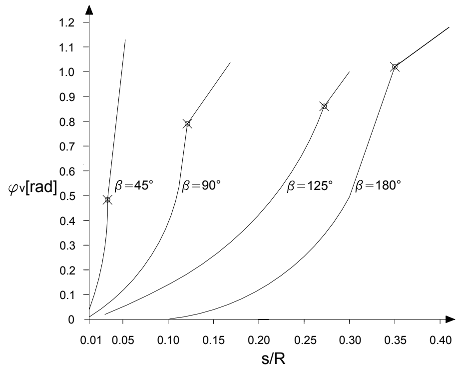

Figure 6: Collapse rotations for circular arches in presence of filling material.

4 EXAMPLE

As an example, the developed procedure is applied to a small railway bridge. The arch is a depressed-arch and has the geometric characteristics given in table 1.

Table 1: Arch characteristics.

Inner span	4.00m
Intrados rise	1.2m
Vault thickness	0.35m
Intrados rise	2.27m
Vault breadth	2.75m
Thickness of the fill above the keystone	1.00m
Masonry specific weight	$18KN/m^3$
Fill specific weight	$18KN/m^3$

The structural capacity of the arch has been evaluated both in presence and in absence of a train in transit. In both cases, the position of the hinges (figure 7) has been evaluated by means of the principle of virtual work.

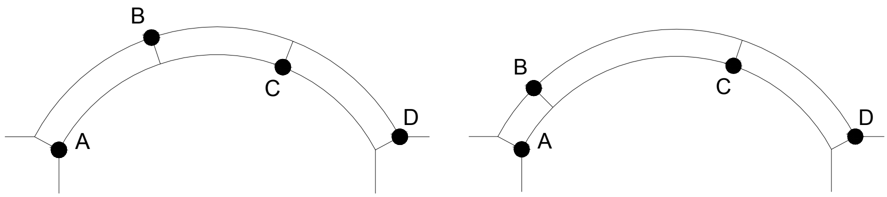

Figure 7: Collapse mechanism with train in transit (a) and without the train (b).

The train axial load is of 250 KN and it is placed at 0.2 times the span from the left springing. The result of the analyses is reported in table 2.

Table 2: Results of the analysis.

	Without the train	With the train
α	0.75g	0.70g
d_u	32cm	35cm

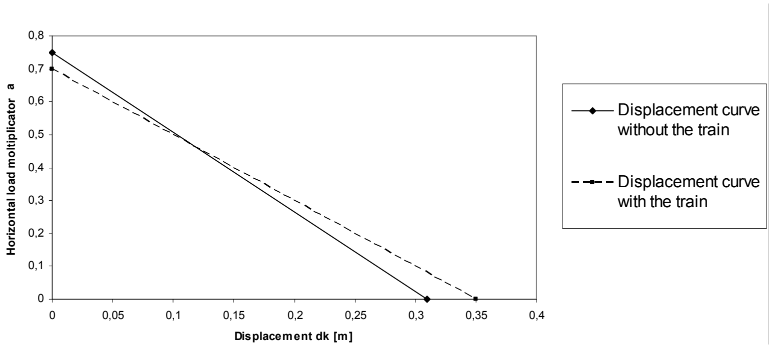

Figure 8: Arch capacity curve.

Figure 8 and Table 2 show that the presence of the train reduces the horizontal load multiplier that activates the collapse mechanism. However, the train in transit produces an increase of the ultimate displacement of the structure at collapse. The different structural capacity found with the train in transit and without the train can be explained as following:

- the virtual work due to the train dead load is positive, therefore, the seismic load needed to activate the kinematic mechanism decreases;

- the collapse of the structure, identified by the ultimate displacement attained by the arch keystone, which is taken as control point of the structure, in both cases occurs when the alignment of the hinges A, B, and C is reached. The presence of the train has an influence only on the position of the hinges. With the train in transit (figure 7a), the displacement of the arch keystone needed to align the hinges A, B, and C, is larger than without the train. The ultimate displacement attained in the two cases varies of about 10%.

5 KINEMATIC MECHANISMS OF ARCH BRIDGES WITH NON INFINITELY RIGID ABUTMENTS

The study of bridges with spans up to 1.5m has also to take into consideration the participation of the abutments into the collapse mechanism, following the same formulation discussed in section 2. In case of bridges with small span, the hinges position is obtained through non-linear finite element models (De Luca, Giordano, Mele, 2004). From the author's results, it emerges that the ratio between the height H and the thickness B of the abutments is the discriminating factor for the collapse modality of the structure. For ratio H/B<1, that is squat abutments, the abutments remain fixed and the local mechanism becomes that already described in section 2 for the masonry arch. If the abutments are slender, the mechanism involves both the arch and the abutments, becoming a global mechanism (figure 9). In some cases it is possible that one abutment remains fixed and only the other one participates to the mechanism.

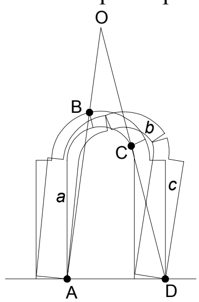

Figure 9: Global mechanism of an arch.

In the implemented procedure, the global mechanism has been studied with the possibility of placing the hinge D (figure 9) at different heights from the ground. The principle of virtual work has been applied to each deformed configuration of the structure, so that it is possible to draw the capacity curve by points and to update in each step the instantaneous position of the rotation centre O. In case that the applied forces remain constant during the development of the mechanism, the capacity curve is linear and decreasing, otherwise it is linear over intervals (OPCM 3431, 2005).

For this particular typology of masonry arch bridges, whose collapse mechanism involves the bridge abutments, also the contribution of the fallow earth thrust and of the active and passive earth thrust mobilized at the limit state, for large displacements, have to be taken into account. The earth wedges behind the abutments, in fact, are able to develop significant forces, that cannot be neglected for a more detailed evaluation of the arch bridge capacity. Their influence on the bridge structural capacity has been evaluated taking into account the following hypotheses.

For bridge abutments built against-earth, the earth thrust is not constant during the development of the mechanism. Before the application of any ground acceleration, the structure is subjected to the fallow earth thrust on both sides. The earth thrust turns into active (on the left abutment) and passive (on the right abutment) when the abutment displacement, due to the application of ground acceleration directed from right leftwards (figure 9), is sufficient to mobilize the earth wedges behind the abutments of the bridge. The threshold between the fallow thrust and the active thrust is quantified in Eurocode 7, (1997).

The change in the capacity curve of a small bridge when the earth thrust turns from fallow thrust to active thrust is shown in figure 10. The first interval of the curves is linear decreasing and starts from a point that represents the horizontal load multiplier that activates the collapse mechanism. At a certain displacement, the capacity curves present a recover of resistance, assuming a pseudo-vertical trend. This happens exactly in correspondence of the change from fallow thrust condition to passive thrust condition on the right abutment. After this recover of resistance, the curves start again decreasing linearly when the forces are stabilized.

The choice of the control point displacement that mobilizes the passive earth thrust can be more or less conservative. In order to analyze the influence of that choice on the final results, it is sufficient to impose different thresholds between the two conditions in the procedure. For example, the displacement of the top of the abutment can be 5cm, 10cm, 15cm and 20cm. Analyzing the four cases (figure 10), it is found that the value of the horizontal load multiplier α after the resistance recovery is not significantly different. Of course, the displacements at which the resistance recovery occurs are different and this can be seen in the curves reported in figure 10.

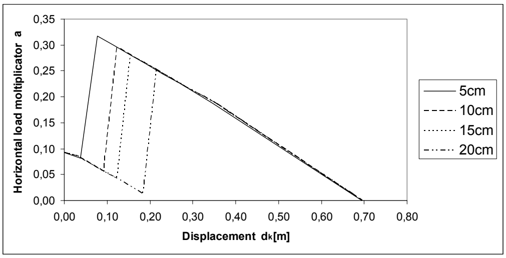

Figure 10: Capacity curve with variable threshold

6 CONCLUSIONS

This work deals with the seismic vulnerability of masonry arch bridge structures. A procedure able not only to recognize the starting condition for the kinematic mechanisms, but also to quantify the ultimate displacement of the structure at collapse has been described. The procedure is based on large displacement hypothesis and gives the limit displacement at collapse for the masonry arch. The achieved numerical results have been validated through a comparison with the results in (Clemente, 1998). A relevant aspect that emerges from the study is the arch collapse

modality, which can change from collapse due to pure equilibrium conditions, typical of round arches or similar, to collapse due to instability of equilibrium, typical of depressed-arches.
A clear distinction has been drawn between bridge structures built with squat abutments (similar to fixed constrains) and structures built with slender abutments (they participate to the collapse mechanism). In the case of slender abutments, a parametric study has been carried out to evaluate the influence of the variation of the earth thrust with the evolving mechanism, obviously in the case that the arch abutments are built against earth. The parametric analysis points out a change in the shape of the capacity curves, even if the mechanism activation and the ultimate displacement remain unaltered. Nevertheless, it is notable that the passive earth thrust generates an upward translation of the curves, starting from the point in which it is mobilized.

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