Assessment of the out-of-plane imperfections of a steel tied arch bridge

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ABSTRACT: One of the decisive parts of the design of a steel tied arch bridge is the buckling strength of the arch. This can be determined by elastic-plastic calculation using finite element software. Assumptions have to be made concerning the geometrical imperfections. An analytical method has been developed to determine the imperfections based on the comparison between stresses derived from strain gauge measurements and calculations. The analytical method is based on the calculation of the stresses caused by all internal forces and the unknown imperfections. The method was validated, using finite element calculations. As the method gives satisfactory results, the stresses resulting from a strain measurements campaign on a real bridge were also used as input for the method. The detailed calculation of the out-of-plane moment resulting from the imperfections is the main contribution of the proposed method.

1 INTRODUCTION

1.1 General overview

For the construction of the high-speed railway line between Antwerp and Amsterdam, a wide variety of slender steel tied arch bridges was built. The buckling behaviour of these arches, which is one of the decisive parts in the design, might be studied by carrying out elastic-plastic simulation using finite element software. However, an important unknown parameter of the problem is the size and shape of the out-of-plane imperfections of the arch.

To assess the size of the imperfections, an analytical model is being proposed, based on a comparison between the calculated stresses in the arch and stresses derived form strain gauge measurements. To deliver these strain values, extensive strain measurements were carried out. Several tied arch bridges were equipped with strain gauges on a number of sections along the entire bridge axis, with unidirectional strain gauges in all four corners of the arch cross-sections. Simultaneously, the stresses resulting from the in-plane bending moment and the thrust force were calculated analytically. The accuracy of the calculated stresses was expanded by incorporating the effect of the transversal bending of the bridge deck. This effect causes an out-of-plane bending moment in the arch as well as a small torsion effect. The measurements pointed out that there was indeed some torsion of the arch section. Despite the fact, that the magnitude of this out-of-plane moment are rather small compared to the effect of the thrust force and the in-plane bending moment, it seems to have an important contribution to the accuracy of the imperfections.

The calculations indicate that a more accurate assessment and better understanding of the stresses, resulting from the effect of the out-of-plane imperfections, might allow for a higher arch carrying capacity. This could then be translated in more elegant arch bridges, which are better accepted by the general public.

1.2 Strain measurements

The analytical model, developed to estimate the out-of-plane imperfections, makes use of stresses, which are derived from strains measured on a real bridge. The bridge, used in this paper to illustrate the method, is the Albert Canal Bridge. This bridge was built in Belgium, near the city of Antwerp, as a part of the new high speed railway. The bridge span equals 115m. The two arches are connected to the lower member by sixteen inclined hangers. The upper bracing is formed by three tubes of large diameters spread out along the length of the arch. The arch springs are tied by the lower chord, consisting of an orthotropic steel deck plate.

Figure 1 : The Albert Canal Bridge.

The bridge was used for extensive strain measurements. One of the two arches was equipped with strain gauges at the four corners of the arch section, and this in 6 sections along the length of the arch. Since the strain gauges were installed after the erection of the bridge, only the live loads were taken into account in the analytical method.

Next to the measured stresses, results from a finite element model were also used as input for the analytical method, in Outtier et al. (2006). This model contained al the details of the bridge, such as the arches diaphragms, the plate connecting the hangers to the arches and the bearings to the smallest detail. The advantage of the use of this finite element model resides in evaluating the analytical model with known imperfections. Indeed, the real imperfections in the arch are unknown to this point.

Another advantage of the finite element model lies in the possibility to distinguish independent actions of the various internal forces.

2 ANALYTICAL METHOD FOR THE CALCULATION OF THE IMPERFECTIONS

2.1 Introduction

The basic idea of the analytical method is that the stresses in a perfect arch are calculated and compared to the stresses in a real arch. The imperfections then follow from the results of this comparison.

The steel tied arch bridge, described above, has a very particular arrangement of the hangers. The inclined arrangement of the hangers, results in a two-dimensional, stable triangular framework, in each arch plane. For the calculation of the normal force and the in-plane bending moment, the arch was thus simplified to a triangular framework. The framework can be calculated by the slope-deflection method. However, once the normal force and in-plane bending moment are known, the curvature of the arch section is taken into account, by superposition of the actual arch behaviour on each arch section.

Since the bearings of the Albert Canal Bridge are not located exactly at the intersection of the axes of the arch and of the lower chord, a supplementary in-plane bending moment is developed in the arch. The horizontal shift of the bearings is necessary, given that the bearing is rather large and additional stiffeners have to be installed in the deck system at the end of the lower chord.

Since the wind bracings resist the out-of-plane bending moment, and assuming that the wind bracings act as fully constrained supports, the out-of-plane bending moment can be easily calculated as for a continuous beam on five supports, in the case of the Albert Canal Bridge.

After the determination of all of the above forces, the out-of-plane bending caused by the unknown imperfections can be calculated.

2.2 Normal force and in-plane bending moment

For the calculation of the normal force and in-plane bending moment, the arch is, as mentioned above, modelled as a triangular framework, consisting of the lower beam, hangers and straight beams between the hangers instead of the curved beams of the actual arch. This framework consists out of 35 beams, connected to each other by 19 nodes. Once the normal force and in-plane bending moment has been calculated, the curvature of the arch will be taken into account.

2.2.1 Slope defection method (triangular framework)

2.2.1.1 Definition of the unknowns

The triangular framework is calculated with the slope-deflection method. The basic conditions are found by expressing rotational equilibrium of each node. In addition the nodes may show displacements, it is necessary to formulate the translation equilibrium besides the rotational equilibrium. Keeping this in mind, there are still 19 unknown node rotations, and 35 unknown rod rotations in the model of the Albert Canal Bridge.

2.2.1.2 Rotational equilibrium

Before writing down the rotational equilibrium of each node, the moments can be determined as a function of the unknown node and beam rotations, using the constitutive equations of the slope defection method, see Eq. (1) and Eq. (2). The sign of each variable is according to the convention to which the moments that rotate a node clockwise or the end of a beam counter clockwise are positive.

$$\overline{M_l} = \overline{M_l^0} - K_i \cdot (2 \cdot \phi_l + \phi_r - 3 \cdot \psi_i) \tag{1}$$

$$\overline{M_r} = \overline{M_r^0} - K_i \cdot (\phi_l + 2 \cdot \phi_r - 3 \cdot \psi_i) \tag{2}$$

Herein, K_i is the stiffness of the beam, ϕ_l and ϕ_r the node rotation respectively at the beginning and the end of the beam and ψ_i the beam rotations.

For each node , the rotational equilibrium is then given by:

$$\sum \overline{M_r} = 0 \tag{3}$$

Equation (3) in combination with equations (1) and (2) gives a system of 19 independent equations in 56 unknown parameters.

2.2.1.3 Translational equilibrium

The translational equilibrium of each node makes use of the normal and shear forces in the rods. The shear forces can easily be found with the help of Eq. (3) and Eq. (4).

$$V_l = -\frac{\overline{M_r} + \overline{M_l}}{l} - V_l^0 \tag{3}$$

$$V_r = -\frac{\overline{M_r} + \overline{M_l}}{l} - V_r^0 \tag{4}$$

The normal forces can however not be determined so easily. According to Vandepitte (1981) the normal forces can be derived from the displacement values of each node, as shown in Fig. 2. The relation between the length l of the rod and the coordinates of the nodes, A and B is given by Eq. (5). Eq. (6) gives then the same relationship between the length $l + \Delta l$ after deformation of the rod and the coordinates after loading A’ and B’. Subtracting Eq. (5) from Eq. (6) gives the normal force as a function of the displacements of the nodes, as can be seen Eq. (7).

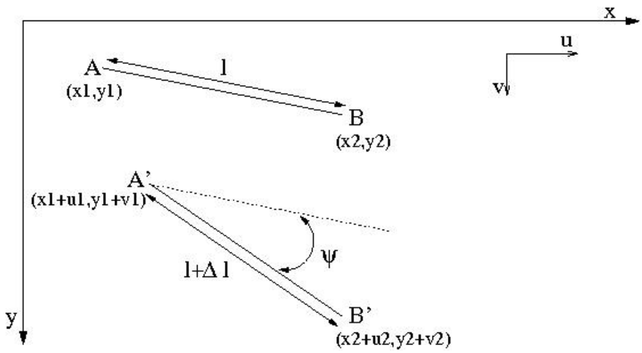

Figure 2 : Displacement of a rod.

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = l^2 \tag{5}$$

$$(x_2 + u_2 - x_1 - u_1)^2 + (y_2 + v_2 - y_1 - v_1)^2 = (l + \Delta l)^2 \tag{6}$$

$$(x_2 - x_1)\cdot(u_2 - u_1) + (y_2 - y_1)\cdot(v_2 - v_1) = l \cdot \Delta l = l \cdot \frac{N}{E \cdot A} \tag{7}$$

Herein, N is the normal force in the beam, A the section of the beam and E the Young's modulus of steel.

Before the normal forces can be entered in the equations for the translation equilibriums for each node of the framework, the displacements of the nodes must still be written as a function of the angular rotations. This can be done in a similar way, as can be seen in Fig. 2 and Eq. (8).

$$\psi = \sqrt{\frac{((y_2 - y_1)\cdot(u_2 - u_1) - (x_2 - x_1)\cdot(v_2 - v_1))^2}{(x_1 - x_2)^2 + (y_1 - y_2)^2}} \tag{8}$$

Since the beam rotations of the hangers are dependent on the displacements of the nodes of the arch and the lower chord, the beam rotations of the hangers can thus be written as a function of the beam rotations of the arch and the lower chord. This reduces the number of unknown parameters to 19 unknown node rotations and 19 unknown beam rotations. In this system, 19 rotational equilibriums can be written, as well as 19 translational equilibriums for each node. Hence the system can be solved.

2.2.1.4 Curvature of the arch

Up to this point, the arch was simplified to straight beams. Based on this simplification, the normal forces in the arch beams are always aligned oriented to axis of the beam, as is standard for the slope deflection method. Nevertheless, because of the curvature of the arch, the normal force has to be oriented according to the tangent to the arch at each point, resulting in a varying direction for the normal force, along the curvature of the arch, see Fig. (3).

This necessitates an adjustment of the normal force and the in-plane bending moment as calculated by the slope-deflection method, given by Eq. (9) and Eq. (10).

$$N' = N \cdot \cos\alpha + V \cdot \sin\alpha \tag{9}$$

$$M' = M - N \cdot y + V \cdot x \tag{10}$$

Herein α is the angle between the tangent to the arch for each point of the arch and the axis of the beam.

The results of these calculations are discussed further on.

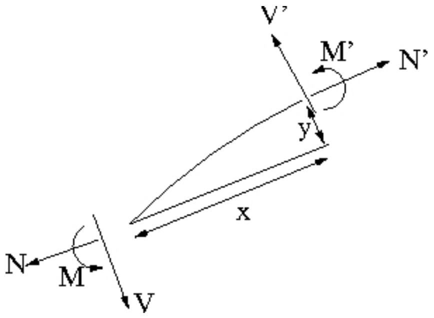

Figure 3: Differences between a straight rod and a curved beam.

2.2.1.5 Supplementary in-plane moment cause by the shift of the bearing

As mentioned before, a supplementary in-plane bending moment exists, caused by the small horizontal shift of the supports. This bending moment equals the resultant of the internal forces in the bridge and the reaction at the bearings, multiplied with the size of the horizontal shift of the bearings.

2.2.1.6 Results

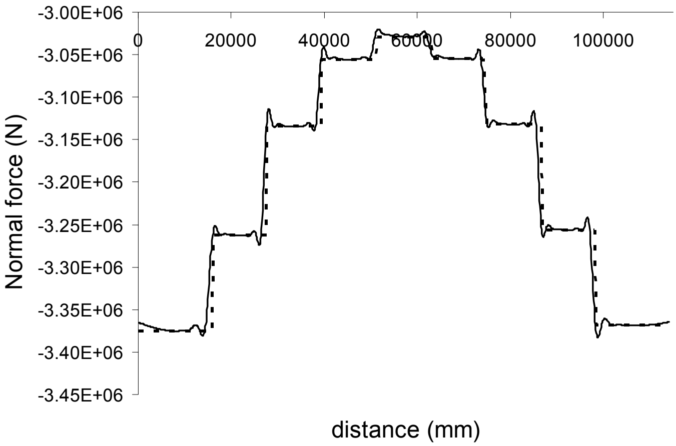

Figure 4: The normal force in the arch (full line) and the normal force in the arch without taking into account the curvature of the arch (dashed line).

The results of the calculations are shown in Fig. 4 and Fig. 5. The dashed lines in Fig. 4 and Fig. 5, represent respectively the normal force and in-plane bending moment, without taking into account the curvature of the arch. If the curvature is taken into account, the full line must be observed.

The difference between the two lines in Fig. 5 is quite important and is the largest just between two points connecting hangers to the arch. Since the measurement locations are situated between these connection points, this adjustment was indispensable for obtaining a more accurate result for the imperfections.

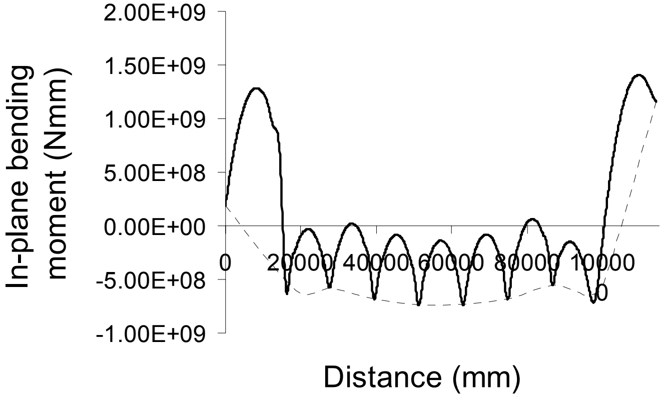

Figure 5: The in-plane bending moment in the arch (full line) and the in-plane bending moment without taking the curvature of the arch into account (dotted line).

2.3 Out-plane bending moment

Disregarding the effect of the wind, the out-of-plane bending moment is primarily caused by the bending of the bridge deck itself, as it introduces a secondary bending moment at the arch springs. Assuming the deck plate is clamped in the arch, the out-of-plane bending moment in the arch can be calculated as a beam on five supports, since there are 3 wind bracings. In reality, the connection between the arch and the arch springs is not fully clamped, so it's situated between a fully clamped connection and hinge connection.

The method also implies the wind bracings to act as rigid supports. It is necessary to taken into account horizontal displacements of the wind bracings, due to the wind. In addition, the live load will cause the arches to move in opposite directions, i.e. towards each other. Hence, the assumption of the bracings acting as fixed supports proves to be valid.

A higher arch stiffness in comparison to the stiffness of the end beam of the bridge deck results in a clamped connection. A hinged connection is achieved when the arch stiffness is much lower. Reality lies somewhere in the middle of both hypotheses. Finite element calculations pointed out that 50% of the moment resulting from a clamped connection might be taken into account.

The out-of-plane bending moment may be determined from the continuous beam analogy, by using the slope deflection method. Since the beam rotations of the continuous beam equal zero, the unknown parameters of the system are the node rotations. The system thus reduces to the rotational equilibrium of the nodes, as was described in paragraph 2.2.1.2.

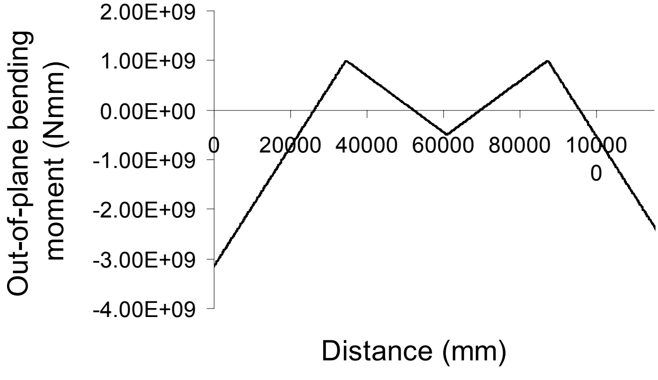

Figure 6: Out-of-plane moment in the arch.

The out-of-plane bending moment is shown in Fig. 6. Despite the fact that the out-of-plane bending moment reaches but a fraction of the in-plane bending moment, it appears of importance to take into account the effect of this moment, since the imperfections will result in a second, supplementary out-of-plane bending moment.

Although, the incorporation of the out-of-plane bending moment in this method might seem somewhat unorthodox, as the bridge deck is flexible when compared to the stiff arch springs, the heavily stiffened edge floor beams introduce the lateral bending by the out-of-plane bending moment. However clamping of arch springs in the edge floor beam is improving the stability.

2.4 Out-of-plane bending moment caused by the imperfections

The normal force acts always at the centroid of the cross section a perfect arch. Hence, in an arch with imperfections, the normal force will act with a certain eccentricity will thus generate a supplementary out-of-plane moment.

If the imperfection would exist in a singular location, acting on an infinitesimal part of the arch, the supplementary out-of-plane moment caused by the normal force is given by Eq. 11.

$$\Delta M = \frac{N \cdot \Delta y \cdot y}{I_z} \qquad (11)$$

Herein Δy is the imperfection, y the distance to the neutral axis and I_z the moment of inertia about the vertical axis.

The moment ΔM influences the internal forces in the whole beam. The additional out-of-plane bending moment reaches a maximal value at the location of the imperfection and fades out nearing the arch ends, as can also be seen in Fig. 7. If there should act a second moment on the beam, the distribution of both moments will influence each other.

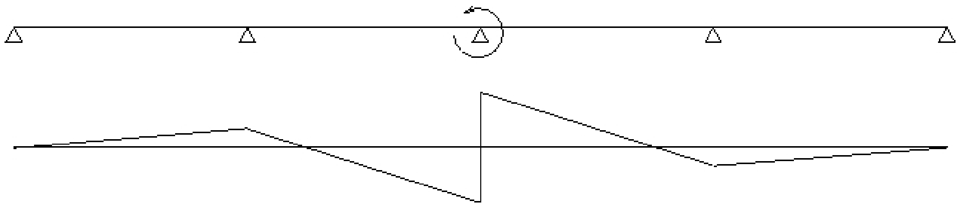

Figure 7: Distribution of the moment in a continuous beam on five supports, when the external loading is only a moment in one single node.

Since the arch shows imperfections across its entire length, and taking into account the above considerations, the effect of the imperfections can be calculated as a uniform moment distribution acting on the arch. At each point of the arch, the normal force shows a certain eccentricity, resulting in out-of-plane bending moments. Given the distribution along the arch of all those moments disturbs each other, the moment resulting from the imperfections can not be calculated using Eq. 11. The resulting moment distribution of all those contributions must be calculated.

The whole method, described in this paper, for determining the imperfections is based on the comparison of the analytical calculated stresses and measured values. Hence, the imperfections in the measurement locations is assumed to be an unknown, and between two measurement locations a linear distribution is accepted between these two values. The accuracy of the method depends on the number of subdivisions taking into account for the calculation of the out-of-plane bending moment, caused by the imperfections.

The calculation of the moment distribution caused by the imperfections can also be done by the slope deflection method. In contrast with the calculation of the out-of-plane bending moment, the continuous beam must be divided in separate beams having external moments at the hinge. Hence, the number of beams in the system will depend on the number of subdivisions of the arch. Thus, the system can be solved by using the rotational and translational equilibriums.

The resulting out-of-plane moment is still a function of the unknown imperfections.

2.5 *Imperfections*

The normal stresses in the four corners of the arch section can be calculated from the normal force, the in-plane bending moment and the out-of-plane bending moment, resulting from the transversal bending of the bridge deck and the imperfections. Since the latter is still a function of the unknown imperfections, the resulting stresses are similar. These stresses can be compared to the measured ones or those derived from the finite element model. The unknown imperfections can then be calculated using this comparison, as shown in Fig. 8.

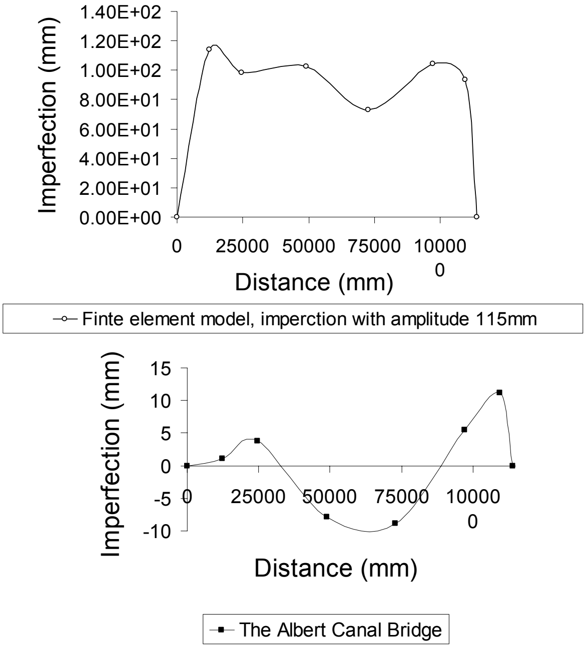

Figure 8: Resulting imperfections.

3 CONCLUSION

An analytical calculation method based on the determination of all internal forces has been proposed. Although the method uses some simplifications, it describes the imperfections rather well. The analytical calculation allows distinguishing between the various effects of the stress resultants. The most important contribution of the method, proposed in this paper, concerns the out-of-plane bending moment caused by the imperfections, which is calculated in detail. However, these imperfections influence the stresses in the entire arch and not only at one point in the arch.

Because of these encouraging results, extensive further research will be undertaken in order to prove in a more general way the relevancy of imperfections and to keep improving these analytical calculations

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